Output 5: Generic Tasks

We have developed a model for generic tasks (blueprints) that one can find everywhere. You can browse through them according to different perspectives.

We created a list of basic fundamental generic tasks taking the national curricula into consideration. This is to address the needs of teachers to understand that they can actually teach their curriculum using MoMaTrE. The generic tasks are described in terms of concepts, types of measures, the expected strategies and results, the competencies that are necessary to complete the task and the objects on which the task can be asked. All partners went through their edited tasks in order to find examples of each generic task and create some in the case that no corresponding example task could be found. All in all, we listed 40 different generic tasks and translated them in each partner language.

Each task is associated with a picture that can be shown in a carousel, or browsed in a catalog, so it is now much easier to create tasks, because there is a wide range of ideas and best-practise examples.

A generic task is not to be confused with a task template. Generic tasks can result in a task template if there is a definite set of measures to be done but not all are as precise as this. For example *modelling a situation with a function* is a wide generic task that has sub-generic tasks, such as *modelling a situation with a quadratic function* that can lead to a specific task template. So the Output O5 is a preparation for some parts of the Output O3.

We see a great innovation in this. So far there is no comparable project, no big collection of math trail tasks. This catalogue is integrated into the web portal, so one can look at the generic tasks, choose one and inside the portal create a local task out of it (sometimes with the help of task templates) and build a math trail with this kind of tasks in a short time and spanning a wide variety of mathematical concepts.

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Since time is one of main issues for teachers not to use a certain technology, we expect this catalogue to have a heavy impact on the number of created math trails and outdoor tasks. Also the transferability potential is big because the generic tasks can inspire teachers to create local tasks or even textbook tasks for normal lessons. It will be the biggest collection of authentic, real-life math tasks so far on the internet.

The following catalogue contains the generic tasks in English with international examples. For the translations of the generic tasks into German, Spanish, French, Slovakian and Portuguese, please visit our website www.momatre.eu!

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Generic Tasks

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GT 1.1 Recognizing a Shape	**Recognize and name a basic shape, model a real object.**
	Data to measure: Planar or 3D-shape s
	Solution : The name of the shapes (as Multiple Choice). In 2D : Polygons (count the number of sides) / Ellipsoid or Hyperbola In 3D : Cuboid, Cone, Cone trunk, Cylinder, Polygonal Prism or Pyramid (see GT 2.5)
	Possible hints: ● Picture based hints (arrows) to locate the shape ● Hints on characteristics of the shape ○ in 2D : “it’s a polygon” / “count the number of side: how do you say that in ancient greek (find other words)?” ○ in 3D : “what are the characteristics of pyramids? prisms? cylinders?”
	Liste d’objets : Buildings (Geometrische Formen, Las Claras, Plaza Mayor en Plase, Comércio Justo), Sculptures (Behind the bust, Figuras en Cruz Pata), Fountains (Formato da Fonte, LAGO), Benches (Banco del Barómetro), Trash can (TUDO LIMPO), Others (Polygones, Drevený totem)
List of referring numbers MCM tasks de: Geometrische Formen; en: Behind the bust; sp: Banco del Barómetro, Las Claras, Plaza Mayor en Plase, Figuras en Cruz Pata; pt: Comércio Justo, Formato da Fonte, TUDO LIMPO, LAGO; fr: Polygones; sl: Drevený totem, Tajomná Vitálišova, Kaplnka na skale, Vojnov-í hrdinovia	

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GT 1.2 Recognizing a symmetry	**Recognize and name the symmetry in a basic shape.**

Curriculum item: level 3 " He (the pupil) recognizes in his environment situations that can be modelled by symmetry (butterflies, buildings)"

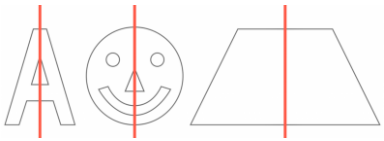
On this picture one can already guess the elements of symmetry : Be careful no to do so.

Data to be measured: Nothing but the observation of the shape.

Solution: Identify the axis of symmetry by naming them, for example horizontal, diagonal, or drawing them (Multiple Choice). Another option: count the number of axes of symmetry (exact value)
NB: A regular polygonal shape with N sides has N axes of symmetry.

Possible Hints:

- Reminder of what is an axis of symmetry. Photo, picture or drawing.



- How many ways do you have to "cut" the whole object in two mirror-objects?
- Be careful to find all the axes and do not count any axis twice.

List of objects: urban art (Space Invaders Space Invader #1), building walls (facade ¿Simetrí-a?, moulded ceiling of an arch Das Deckengewölbe, sculpted antic wall in a museum Figura circular), Street container (octogonal fountain Fountain of justice), Windows (stain glass rose of a church Catedral Vieja de Pl), Mosaics (Mural fresco História nos azulej, Pavements Pedra na calçada)

List of objects: urban art (Space Invaders Space Invader #1), building walls (facade ¿Simetrí-a?, moulded ceiling of an arch Das Deckengewölbe, sculpted antic wall in a museum Figura circular), Street container (octogonal fountain Fountain of justice), Windows (stain glass rose of a church Catedral Vieja de Pl), Mosaics (Mural fresco História nos azulej, Pavements Pedra na calçada)

List of referring numbers MCM tasks de:Das Deckengewölbe; en:Fountain of justice; sp: ¿Simetrí-a?, Figura circular, Catedral Vieja de Pl; pt: História nos azulej, Pedra na calçada; fr: Space Invader #1;

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GT 2.1 Enumeration	**Identify the elements of a simple collection and enumerate it.**
	Data to be measured: The total number of objects. Solution: The number by direct count, finding regularities or an area: 1. $N = A \cdot n$ with A being the total area and n the number of objects per m². 2. In a rectangular disposition, one multiplies the number of rows and lines. Possible Hints: ● How many xx are in one row? ● How many xx can you count? ● How do you calculate the number of pieces in a puzzle? ● Can you identify regularities? ● Determine the number of xx in a special area, e.g. 1m² or 4m². ● Consider how often this area will fit into the total area.
List of objects Bus stops (Buslinie 13, Ist der Bus schon we), Stairs (Les marches de la co), Buildings (Courtyard of the Rec,San Saturio, junto a, Lutero se asoma a la, Ventanas del colegio), Floor (Identificando cuadra, Número de lajes), Stone wall (Murinho), Sculptures (Parábola del Ángel), Benches (Bancos), Arts (Les travailleurs)	
Wizard Stones, Advertisement Pillar	
List of referring numbers MCM tasks de:Buslinie 13, Ist der Bus schon we; en: Courtyard of the Rec; sp: Identificando cuadra, San Saturio, junto a, Lutero se asoma a la, Parábola del Ángel, Ventanas del colegio ; pt: Bancos, Murinho, Número de lajes; fr: Les travailleurs; Les marches de la connaissance	

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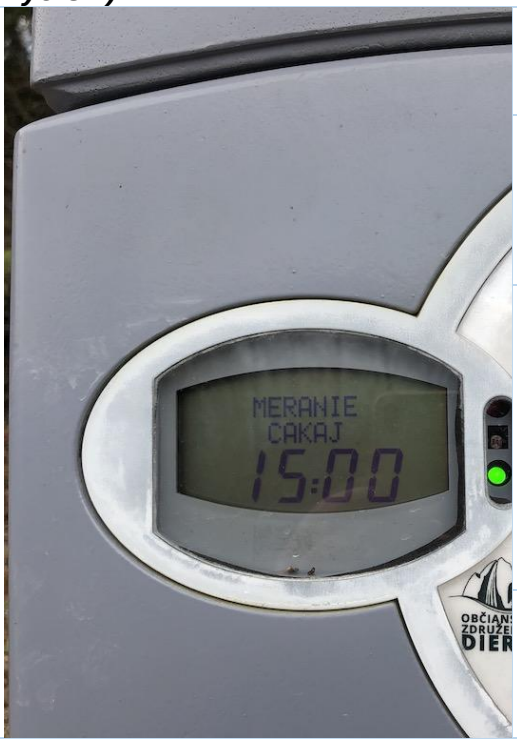
GT 2.2 Information	Recognizing and computing with numbers or symbols.

Curriculum item: level 2 "The pupil finds, recognizes and understands the relation between a number and object. The pupil identifies and interprets with the help of data and simple numerical text messages from the school and family environment (schedules, shifts, publicity flyers...)"



Data to be measured:
Numbers in various number systems (Arabic, Roman, sexagesimal like digital clock…) in particular by analysing numbers table; parking automat payment; opening hours of the institution; memorial, information table analysis.

Solution: The main solution is based on using information and then recognize, record and/or do some arithmetic or translations/transformations with (units recognition and/or substitution).
Recognition of the number(s), transformation to the other (given) number system; arithmetic operation: counting age of he person on the statue, counting number of years to/from some event(s), numbers or results comparison; recognizing the number upon the given information, finding the object with the given number, etc.

Possible Hints:

- Be careful in the number(s) recording.
- Record and work with number(s) of given properties, if there are more numbers on the object displayed or to find. Given properties are: digits of the number, size of the number (e.g. consider number(s) larger than …); even number, odd number, decimal number, ...
- Clarifying question about the situation; e.g. How many hours do you want to park? Which object are you analysing?
- Roman numerals coding; transformation of units;
- Clarifying photos.

List of objects **:** parking automat (Achtung, Strafzettel), information table (Roman Numerals, Plaza del Cabildo, Počet cyklotrás), memorial table (Pamätná tabula) , object (trees, wardrobe boxes) label (Magnólia, Wardrobe boxes), object attribute (book page) (De revolutionibus, Les décimales), map (Nadmorská výš¡ka)

List of referring numbers MCM tasks
de: Achtung, Strafzettel; en: Roman Numerals, Wardrobe boxes; sp: Plaza del Cabildo; pt: Magnólia; fr: De revolutionibus, Les décimales, Der Fluss, 100 000 documents!; sl: Nadmorská výš¡ka, Počet cyklotrás, Pamätná tabula

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GT 2.3 Number of rectangular objects, regularly arranged	**Decompose a collection of rectangular objects to compute their number by multiplication, addition and/or substraction**
Curriculum item: level 2 "The pupil finds, recognizes and counts the rectangular objects, which are regularly arranged." "To identify and differentiate rectangular shapes in school and family environments by describing them using basic vocabulary."	
	Data to be measured: The number of one type of rectangular objects (window, window glass, tile, stone, plate, brick) that are horizontally and vertically arranged. The objects cover a precisely defined area (pavement, floor, roof, wall or other surface). The shape of the area covered by the objects can be regular or irregular; some objects can be missing or added.
	Solution: Simple counting objects by rows and columns, one by one, counting aligned objects. Expert strategy: multiplication followed by subtraction or addition. The result is the product plus or minus extra or missing objects. This requires to complete the area by adding or subtracting a proper number of the same objects. For example counting the height of the building using the number of bricks or plates on the wall.
	Possible Hints: ● Be careful with the area. ● Be careful with the number of lines and rows of the assigned area. ● Clarifying question about the situation; e.g. How many tiles are missing? ● Clarifying shape and type of the object by photo or picture.
	List of objects: parts of window glass (Transparente opacité, Fenster, Windows), objects arranged in grid or matrix (Anzahl der Löcher), metal roof, plastic object (Castillo en el parque), pedestrian zone pavement (Pavimento), building façade (Height of the building, Edificio Portico), number of doors of boxes arranged in matrix (Combien de casiers?), stain-glass object, windows arranged in matrix (Numero de janelas, Sklíčka)
List of referring numbers MCM tasks de: Fenster, Anzahl der Löcher; en: Height of the building; sp: Castillo en el parque, Edificio Portico; pt: Pavimento, Numero de janelas; fr: Combien de casiers?, Transparente opacité; sl: Windows, Sklíčka	

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GT 2.4 Frequency, Fraction	**Calculate the ratio of a number/size by the total number/size with n decimals**
Curriculum item : level 9 " The pupil computes sizes and frequencies (of populations)"	
	Data to be measured: Ratio of integers : Number of special occurrences (size of the sub-population) and the total number of elements (size of the whole population) Ratio of measures (length, areas, volumes, durations)
	Solution: Count/calculate the size of each population and divide the size of the subpopulation by the size of the whole one. Round the result to the number of decimals that is asked for or give it in percentage.
	Possible Hints: ● Formula for frequency/probability : $\frac{number/size\ of\ desired\ sub\ population/object}{number/size\ of\ total\ population/object}$ ● Formulas for areas : area of a disc from the radius R = π*R^2, area of triangles from the measures of both the basis B and the height H = B*H/2 ● Remember to round the result with n decimals ● Notice symmetries if there are any in order to simplify measures/calculations
	List of objects ● For ratios of integers: bricks/tiles on walls/roofs (high school building Farbenspiel, highschool lockers Le casier noir), letters/sentences in a text (alphanumeric clock Relative Vokale, epigraph/poem/hymn Himno Gallego) ● For ratios of measures: areas on the ground (Concentric shapes Weiss und Grün, Das Runde muss ins Eckige , Puntos cardinales), areas of vertical shapes (triangular sculpture Proporção de triângulos), radiuses of objects (tree Proporciones en la Araucaria excelsa), probability in time (green traffic light SEMÁFORO)
List of referring numbers MCM tasks de: Weiss und Grün, Farbenspiel, Relative Vokale, Relative Vokale en: sp: Himno Gallego, Proporciones en la Araucaria excelsa, Puntos cardinales pt: Proporção de triângulos, SEMÁFORO fr Le casier noir	

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GT 2.5 Polyhedron.	**Polyhedron; Determine the number of faces, edges or vertices. Recognize a regular polyhedron.**

Curriculum item: level 4 "The pupil understands polyhedron as a solid figure bounded by some number of plane polygonal faces. The pupil identifies and counts assigned elements of polyhedra." "To recognize and identify polyhedrons, prisms, pyramids, in the immediate environment by describing their basic elements."

Data to be measured: The number of polygonal faces, edges, vertices or degree of vertices of the polyhedron. Axes of symmetry.

Solution: Recognizing the polyhedron (Multiple Choice) or counting some features (Exact number of faces, their number of sides, edges, vertices or their degree, or a combination of those like the Euler constant F-E+V=2 for a topological sphere). Simple counting the polygonal faces (same shape) of the polyhedron. Recognizing regular polyhedra, Platonic solids: regular tetrahedron (4 triangular faces, 6 edges, 4 vertices of degree 3), cube (6 square faces, 12 edges, 8 vertices of degree 3), regular octahedron (8 triangular faces, 12 edges, 6 vertices of degree 4), regular dodecahedron (12 pentagonal faces, 30 edges, 20 vertices of degree 3), regular icosahedron (20 triangular faces, 30 edges, 12 vertices of degree 5). Result: Name (Multiple Choice) or number (Exact value) of faces (F), edges (E) or vertices (V). Discover or verify Euler's Theorem for polyhedra: V – E + F = 2 (Exact value).
This task implies symmetries understanding, enumeration strategy elaboration.
It could be elaborated by counting more specific quantities associated with the polyhedral object or its parts, like area of faces, total area, length of edges, total height or diameter...

Possible Hints:

- Clarifying question about the situation; e.g. How many polyhedral elements are not visible?
- Clarifying shape and type of the object by photo or picture.
- Ancient Greek: Tetra = 4, Penta = 5, Hexa = 6, Octo = 8, Dodeca = 12, Icosa = 20, Edra = face.

List of objects: stairs (Treppenstufen beim G), art object composed by polyhedral (Torre poliédrica, Poriadok musí byť, Atrás do busto), signalling (Biblioteca), statue, models of Platonic solids displayed in the outdoor area (Platón visita Madrid), irregular polyhedron (Štvorsten).

List of referring numbers MCM tasks
de: Treppenstufen beim Gästehaus; en: Many faces for a crystal; sp Torre poliédrica, Platón visita Madrid; pt: Atrás do busto, Biblioteca; fr: Le cristal, de face et de profil, Sans perdre la face; sl: Poriadok musí byť, Štvorsten

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GT 2.6 Combinatorics	**Find the number of specified arrangements or permutations of people/objects**
Curriculum item: level 9 " Enumeration with the help of tables and trees"	
 *How many possible combinations exist for arranging the flags over the principal entrance (there are several posts not seen on the picture)?*	Data to be measured: Identify a configuration space and compute the number of its elements e.g. Bike Stands: One has to count the number of bike stands and whether it is possible to lock one or two bikes at each stand. It is important that one cannot already see the total number of stands in the picture. Solution: Come up with an enumeration strategy and compute the number of configurations. e.g. Bike Stands: For the first bike, one has n possibilities, for the second bike n-1 and for bike number k: n-(k+1). These possibilities have to be multiplied. This leads to the following calculation: $N= n\cdot(n-1)\cdot(n-2)\cdot\ .\ .\ .\cdot(n-(k+1))$ possibilities. One can further solve the task with help of the formula of the urn problem "with sequence without repetition". The n balls in the urn symbolize the places at the bike stands. The first bike is assigned to the first drawn ball, the second bike to the second drawn ball, and bike number k to ball number k. The number of possibilities is $N=n!/(n-k)!$ Possible Hints: ● Can the space of configuration be split into a product of two sub-configurations (ex colours x forms, rows x lines) ● Notice symmetries if there are any in order to simplify computations ● The number of permutations is n!=nx(n-1)x...x2 combinations of choosing ● How many possibilities exist for the first object to be placed?

List of objects

- Permutations : bike stands (Bike Stand in Zaryad, Soporte de bicicleta, Bicicletas, ESTACIONAR A BICICLE, Na bicykli do Coop J, Bicycles, Bicicletas or skateboard stands Les skate-boards amoureux), boat dock (Embarcadero Puertoch), train platforms (Destinos dos comboio), chairs/benches (in a lobby Posibilidades de sen, around a statue COMO SENTAR, a square (Bench combinatorics, Lavicky a páry, a fountain Posedenie pri fontá, in a field Posedenie pri ohné-k), flags (on a building Flags, on a square), engraved text (Vlajky na mestskom é)

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List of referring numbers MCM tasks

de:Fahrradparkplatz, Kombiniere Dich ins Mausoleum, Treppenstufen im Stadtpark; en: Bike Stand in Zaryad, Bicycles, Bench combinatorics, Stairs in front of the town hall, Flags; sp: Soporte de bicicleta, Posibilidades de sen, Embarcadero Puertoch; pt: Bicicletas, Destinos dos comboio, Painel de grafite, COMO SENTAR, ESTACIONAR A BICICLE, Bicicletas; fr: Les skate-boards amo, Les skate-boards amoureux, Les nouvelles tables de l'INSPE, Une disposition particulière, Coloriage; sl: Vlajky na mestskom é, Lavicky a páry, Schody ku spomienkam, Schodisko, Posedenie pri fontá, Posedenie pri ohné-k, šmýkacka, Na bicykli do Coop J, Obecný úrad, Hotel TATRA

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GT 2.7 Sequences	**Use sequences to model a phenomenon**

Curriculum item: (in Spain) level 11 (17 years old). *"To solve problems linked to real situations in which it is necessary to identify numerical successions and progressions", "To obtain and manipulate symbolic expressions that describe numerical successions, observing regularities in simple cases that include recursive patterns".*

Data to be measured: Identify a sequence, the number of occurrences/possibilities, ratio between terms of a sequence. In this example: going up the stairs, you can climb one or two steps at a time. How many different ways can you go up the stairs?

Solution: The value of the sequence at a given step. Understand a series of events, model it with a mathematical sequence, compute its value. There are different possibilities to solve the task: One possibility is to note down the possibilities systematically.
In the stairs example for n= 5: (1 1 1 1 1) (1 Possibility), (2 1 1 1) (4 Possibilities), (2 2 1) (3 Possibilities), so there are 8 possibilities. Another possibility is to use the Fibonacci numbers. According to this, the number of possibilities to walk a stair with n steps equals the number of possibilities to walk stairs with (n-1) + (n-2) steps.
The sequence is: (1) 1 2 3 5 8 13 21 34 55 89 etc.

Possible Hints:

- Try to solve the problem starting with one step, then two, then three and find the pattern.
- Relation with a certain type of sequence such as the Fibonacci sequence.

List of objects: stairs (Stairs in Zaryadye Parc, Stairs, Stairs, Stairs, Stairs in front of t, Stairs, Stairs, L'ascension, Stairs, Stairs, Stairs, Escaleras Centro Bot, Stairs, Stairs, Stairs in Front of B, Stairs, Stairs), pinecone (PIÃAS DE FIBONACCI), sanitation cover (Padroes na tampa de), tiles (Ninho de retangulos) Fibonacci : stairs (Kombiniere Dich ins Mausoleum, Treppenstufen im Stadtpark, Stairs in front of town hall, Schody ku spomienkam, Schodisko, Šmýkačka, Obecný úrad)

List of referring numbers MCM tasks: de: en: Stairs, Stairs, Stairs, Stairs, Stairs, Stairs, Stairs, Stairs, Stairs; sp: PIÃAS DE FIBONACCI, Escaleras Centro Bot; pt: Padroes na tampa de, Ninho de retangulos; fr: L'ascension;

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GT 3.1 Simple straight line	**Measure or calculate the distance between two points (segment)**

Curriculum item: (in Spain) level 5 (11 years old). *"To make approximate measurements of lengths or distances, using the appropriate tools (ruler, tape measures...) and to express the result of the measurement using the most appropriate units", "To calculate real distances between points in the plane"* **BEWARE that GT3.7 deals more specifically with heights**

Data to be measured: The distance between two points

Solution: To measure directly or indirectly with a ruler or to find a unit pattern (step, slab, pavement stone) to compute the distance between two points.
Choose a suitable tool adapted to the size (steps, string, ruler…).
Bring closer an unreachable length into a reachable one by a transformation: reduction, similarity, homothety, rotation.
Use repetition of a pattern to fall back to counting integer numbers and multiplication by the measure of the unit.

Possible Hints:

- If you don't have a ruler, use a unit pattern
- Maybe you can count using feet (steps) or hands
- If the distance is a height (vertical), find a similar horizontal distance easier to measure
- Use Pythagoras Theorem for hypotenuse of a square triangle..

List of objects: pavement (Bordstein), Palace (La longitud del pala), arch (Arco del Parque), facade (Plaza Mayor - 3), handrails (Drahtsicherung), stairs (Subida al castillo), fountain (Peré-metro da Fonte), building (Altura do edifé-cio), tree (La hauteur de la sculpture)

List of referring numbers MCM tasks: de:Bordstein, Drahtsicherung; en: How high is the tree?, Distance; it: La dimensione; sp: La longitud del pala, Arco del Parque, Plaza Mayor - 3, Subida al castillo; pt: Peré-metro da Fonte, Altura do edifé-cio; fr: La hauteur de la sculpture

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GT 3.3 Diameter of a circle	**Estimate the diameter, radius or perimeter of a circle**
Curriculum item: (in Spain) level 6 (12 years old) Geometry Module 1º ESO *"To calculate the length of the circumference of a circle."*	
	Data to be measured: The radius (r) or diameter (D) or perimeter (P) of the circle.
	Solution: Measure directly the diameter, or the perimeter, or the radius, and use the formula P=πD=2πr
	Possible Hints: ● Use a unit pattern to measure the perimeter ● Use steps to measure ● Use a cord or a suitable tool to measure ● Use the formula P=πD=2πr
	List of objects: pot (Small forest in the), tree trunk (Die alte Linde), benches (El número π), fountain (Fuente concentrica), outdoor gym (¿Cuánto metal?), sculpture (Pedra esculpida)
List of referring numbers MCM tasks: de: Die alte Linde ; en: Small forest ; sp: El número π , ¿Cuánto metal? ; pt: Pedra esculpida ; fr: Diamètre	

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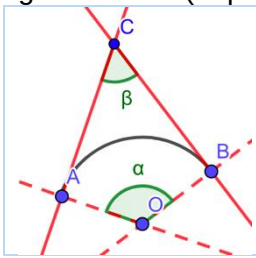
GT 3.4 Radius of a circular arc	**Relate the radius of a circular arc from its length and its center angle**

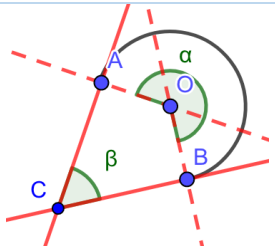
Curriculum item : level 10 " Calculate length in complex figures."

Data to be measured: (NB the center of the circular arc should not be reachable)
- The angle β in degrees between the two tangents of the arc at his extreme points (protractor)
- The length L of the circular arc (with a softruler)

Solution: The center angle α of the circular arc can be determine from the angle β like in the figures below (depending on whether the arc is more or less than half of the whole circle) :



$$\alpha = 180° - \beta$$



$$\alpha = 180° + \beta$$

Then, by proportionality, the perimeter of the whole circle is $P = L \times \frac{360}{\alpha}$

And its radius is $R = P/2\pi$

Possible Hints/subtasks:

- Draw a sketch of the situation and draw the tangents at the extreme points
- The center angle of the circular arc depends on the angle of the two tangents
- The length of the arc and the center angle are proportional
- The formula giving the perimeter P of a whole circle is $P = 2\pi \times R$

List of objects **:** Street or park water pools, the outside of rounded buildings with big walls (theatres, churches or cathedrals, market place)

List of referring numbers MCM tasks de: Flags, Sitzkreis-Radius; en: 395287; sp: Puerta del Convento, Semicircunferencias; pt: Coroa circular; fr: Cercle dans la prairie, Le rayon de CPE, Le diamètre;

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GT 3.5 Cartesian coordinates	**Situate an object in an axis system by its 2 coordinates (measured or reported)**
Curriculum item : level 4 " finds ones position on a map or a grid"	
	Data to be measured: Numbers of horizontal and vertical movements on the grid to get from the origin to the correct location (a point or a square on a grid) Solution: From the down left corner (or the mentioned origin) of the grid/map, Count the number horizontal (resp. vertical) moves to get the horizontal (resp. vertical) coordinate of the location on the grid/map Possible Hints: ● The horizontal/vertical coordinate is the number of moves one has to do to get to the location from the down left corner of the grid/map ● Count the number of moves to the left or right from this corner (horizontal coordinate) and the number of moves to the top or bottom (vertical coordinate)
List of objects **:** Windows of a building (Karton-Fenster), Squared Map (of a city, a school, a museum,…), chess board in a park, wooden panel, grid or net, net of ropes ladder on playground (Mrieška - súradnic)	
List of referring numbers MCM tasks de: Karton-Fenster, Koordinaten, Koordinaten folgen; en:; sp: Coordenadas, Función Lineal; pt: Janelas centrais ; fr: Au Louvre...B404, L'abscisse d'homo; sl: Mrieška - súradnic, Mriežka - obdĺžniky	

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GT 3.6 Scale	**Determine the scale of this map/model.** **Determine the size of the real object, knowing the scale of a model.**

Curriculum item: level 8 "To calculate real dimensions of length and surface measurements in similar situations: plans, maps, aerial photos, etc."

Data to be measured:
Real length L Model length l
Do not forget to use the same units in both measures.

Solution:
The ratio R is $R = \frac{L}{l}$
The scale is 1: R. One can ask whether for R (large) of for the ratio 1/R (small). The second one is the usual meaning.

Possible Hints:

- A scale is usually given in the form 1:x. This means that 1 cm on the card corresponds to x cm in reality. Beware of the units, 1/x is a very small number!
- Find a striking figure on the map and look for the original in the area.
- Measure one side of the figure on the plan and the one it corresponds to in reality.
- Use the scale to calculate the length of the real object from the length of the model object.
- The scale is 1:x. Multiply the length of the model object by x to get the length of the real object.

List of objects Map (Massstab, La escala de la Magd, La escala, Escala, Percorrer o parque, L'échelle qui ne monte pas, Beš¾ecká trasa pri, Mierka informaÄnej), Sculpture (Schrumpfung), Paint mural (Seminario Padres Agu), Model (Bronzestadt, The scale model), World map (La Escala del Mundo)

List of referring numbers MCM tasks: de: Maéssstab, Schrumpfung, Bronzestadt; en: The scale model, Scale of the torch model, The scale of the map; sp: La escala de la Magd, La escala, Seminario Padres Agu, La Escala del Mundo; pt: Escala, Percorrer o parque; fr: L'échelle qui ne monte pas, Echelle d'un plan; sl: Beš¾ecká trasa pri, Mierka informanej

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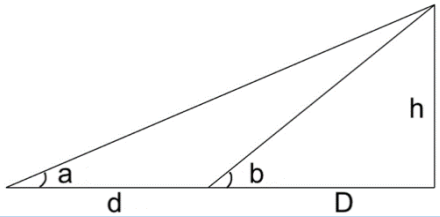
GT 3.7 Height	**Determine the height of an object.**

Curriculum item: level 10: "To calculate real dimensions of length measurements. To recognize similar triangles and, in similar situations, use the theorem of Thales for the indirect calculation of lengths in different contexts."

Data to be measured:
An accessible height h and number of equal objects n
OR stick height h and shadow length s, shadow length of the object S
OR two angles of elevation α and β, distance d between the two points

Solution:

1. The object can be measured using n times a given smaller object of accessible height h. Measuring the height of this smaller object and counting how many times (n) it fits into what we intend to measure, we obtain:
$$H = n \cdot h$$
2. Using Intercept Theorem (Thales' Theorem), by measuring the shadow of a stick or a person, and measuring the shadow of the object S, we get:
$$H = \frac{S}{s} \cdot h$$
3. Solve the system of linear equations:
$$\begin{cases} \tan(a) = \dfrac{h}{D+d} \\ \tan(b) = \dfrac{h}{D} \end{cases}$$



Possible Hints:

- Search for regularities on the object you intend to measure.
- Count the stones (tiles, etc.) in a row from floor to roof. How high are the stones?
- Use this height and the total number of stones to calculate the total size of the object.

OR

- Find similar triangles. Help yourself from a stick or someone else.
- Can you measure the shadow of the object you intend to measure? Also measure the shadow of the stick or of your friend.

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	OR • Can you not measure the shadow? Place your friend in the Intercept position (Tales' position) with the tower. • Move to a flat and wide area that allows you to get away from the object you intend to measure. Measure the elevation angle at the end of the object. • Approach the object a few meters (keep an eye on it!) and measure the angle again. • With these data, solve the trigonometric problem to estimate the height.
List of objects Towers (Turmbau zu Gmünd, Alcoholera, De Madrid al Cielo), Buildings (Schulgebäude des Parler-Gymnasiums, Haushöhe, Height of the ice cave, ALTURA do PRéDIO), Trees (A tree in Azorin, La altura del ciprés), Lamps (Height of the lamp, La altura de una farol), Columns (Height of a column), Sculptures (TORRE de CUBOS), Roof (La hauteur du grenie)	
List of referring numbers MCM tasks de: Turmbau zu Gmünd, Schulgebäude des Parler-Gymnasiums, Haushöhe en: Height of the ice cave, A tree in Azorin, Height of the lamp, Height of a column sp: Alcoholera, La altura del ciprés, De Madrid al Cielo, La altura de una farol pt: TORRE de CUBOS, ALTURA do PRéDIO fr: La hauteur du grenie	

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GT 4.1 Slope	**Determine the slope of a climb.** **Give the result in percentage (or degrees).**
 *To make sure that ramps can be used by wheelchairs, their slope must not be higher than 6% - this could be examined in a MCM task as well.*	Data to be measured: Difference in height Δy; Difference in length (horizontal) Δx or Length of the ramp Δl Solution: $m = \frac{\Delta y}{\Delta x}$ or $m = \frac{\Delta y}{\sqrt{(\Delta l)^2 - (\Delta y)^2}}$ For the degree of the slope: $\alpha = tan^{-1}\left(\frac{\Delta y}{\Delta x}\right)$ or $\alpha = sin^{-1}\left(\frac{\Delta y}{\Delta l}\right)$ Possible Hints: ● Use a gradient triangle. ● $m = \frac{\Delta y}{\Delta x}$: Difference in height divided by difference in length (horizontal) ● The result should be given in percentage. For example, m=0,8 equals 80 percent. For results in degrees: ● Give the result in degrees, $tan^{-1}(m) = \alpha$. Be sure that your calculator mode is on DEG. List of objects Ramps (Slope of the ramp, Rampa para bicicleta, RAMPA DE ACESSO, Skate park, Sklon nájazdovej ra, Ide sa do múzea), Handrails (Steigung des Handlaufs), Stairs Pendiente de las escaleras), Slides (Pendiente del tobogé)
Wizard Slope of a Ramp/Slope of a Handrail in Percentage/Degrees	
List of referring numbers MCM tasks de: Steigung des Handlaufs; en: Slope of the ramp; sp: Pendiente de las escaleras, Rampa para bicicleta, Pendiente de las escaleras, Pendiente del tobogé; pt: RAMPA DE ACESSO; fr: Au Louvre...B404; sl: Skate park, Sklon nájazdovej ra, Ide sa do múzea	

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GT 4.2 Long slope	**Calculate the angle between a slope and the horizontal**
Curriculum item: level 8 " In a right triangle, he (the pupil) uses cosine to determine the measure of an angle"	
	Data to be measured: The elevation *h* and the (possibly curved) length *L* between to distant points of the slope *NB : the use of the protractor should be less precise than the use of the measures below + one needs a calculator to inverse the cosine*
	Solution: Calculate the top angle α by the use of the inverse of the cosine function $\alpha = Acos(h/L)$ in degrees The angle of the slope is the complementary angle 90°- α
	Possible Hints/subtasks: ● The cosine of an angle in a right triangle is the ratio between the adjacent length and the hypotenuse. ● Find the angle between the slope and the vertical using the inverse of cosine function. ● In a right triangle, the two other angles sum up to 90°.
List of objects **:** road (Rampensteigung am Rhein), inclined wall (Pavilhão da égua), stairs (Wendeltreppe, Grau de inclinação), curved ramps for people with handicap (La pente de l'audito)	
List of referring numbers MCM tasks de: Rampensteigung am Rhein, Wendeltreppe; en: sp: pt: Pavilhão da égua, Arranjo do relógio, Grau de inclinação; fr: La pente de l'audito	

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GT 4.3 Measure Angle	**Directly measure an angle**
	Data to be measured: The angle between two definite lines, for example formed by the triangle of your position and two remote targets
	Solution: Identify the two lines, whether - draw a sketch on a horizontal sheet of paper by aiming at the two targets, - construct a paper folding replica of the angle, - estimate the fraction of the flat angle it makes by duplicating the angle, use protractor or trigonometry to estimate the angle. - estimate the angle by means of a Jacob's staff or a similar sticks contraption, based on the $2\ atan(w/2/r) \simeq w/r$ formula.
	Possible Hints/subtasks: ● Would it be easier if laid on paper? How can you make a faithful sketch of the situation? ● [For an obtuse angle] Try to fold a paper sheet that fits the angle.
List of objects **:** Circular tilling (around trees Winkel), wheels (sculpture Bienvenidos al IES N, old mill La noria de la roton), blocks of circular buildings (Casa de la India), Pole (Hauteur de Charpenne)	
List of referring numbers MCM tasks de: Winkel, Stadtpanorama en: sp: Bienvenidos al IES N, Reloj floral, Casa de la India, La noria de la roton pt: fr: Angle de vision d'un, L'angle de la fenêt	

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GT 4.4 Compute Angle	**Calculate an angle by a portion of a circle perimeter**
Curriculum item: level 6 "The pupil knows the formula of the perimeter of a circle and uses it" "resolves problems of proportionality" "knows how to measure or realize an angle of a given size"	
	Data to be measured: The length *L* of the portion of a circle perimeter. The radius *r* of this circle.
	Solution: Identify the center of the circle, measure several radii (if they are not close enough, you may be wrong on the spot of the center) The angle $\alpha = L/r$ in radian, $\alpha = 180 * (L/r)/\pi$ in degrees. If the radius *r* is much greater than the *L*, this formula is approximately the same as the arctan formula in a triangle. So this can as well apply to thin triangles.
	Possible Hints/subtasks: • Where is the center of the circle? • What is the radius r of the circle? • What is the length L of the portion of perimeter? • The angle $\alpha = L/r$
List of objects **:** Circular tilling (around trees Winkel), wheels (sculpture Bienvenidos al IES N, old mill La noria de la roton), blocks of circular buildings (Casa de la India, windows of a circular tower L'angle de la fenêt), FoV field of view (camera Angle de vision d'un)	
List of referring numbers MCM tasks de: Winkel, Stadtpanorama; en: sp: Bienvenidos al IES N, Reloj floral, Casa de la India, La noria de la roton; pt: fr: Angle de vision d'un, L'angle de la fenêt;	

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GT 5.1 Simple rectangle area	**Determine the area of an object of rectangular shape.**

Curriculum item: level 4 (10 years old): "To calculate the perimeter and the area of polygons in context problems by applying appropriate formulas and techniques."

Data to be measured:
Length l Width w
The data should be measured several times at different locations to consider deviations from the chosen model.
Do not forget to use the same units in both measurements.

Solution: $A = l \cdot w$

Possible Hints:

- Which shape models the object?
- You can assume that it is a rectangle. Determine its area.
- Area = length multiplied by width. Beware of the units!

List of objects Flowers bed (Blumenbeet), Goal (Área de una portería), Board (Por el Juan Carlos I, PRESERVA A NATUREZA, Reklamný pútaÄ, Oprava nástenky, Knihobúdka), Building base (2. Longitudes Capill), Furniture (Qué-mica mineral), Parallelepiped surface (Casota), Bridge (A PONTE É UMA PASSA), Table (Ping-pongový stôl), Tennis court (Tenisový kurt)

List of referring numbers MCM tasks de: Blumenbeet, 261622; en: 181041; sp: érea de una porteré, Por el Juan Carlos I, 2. Longitudes Capill, Qué-mica mineral; pt: Casota, Todos os patinhos..., PRESERVA A NATUREZA, A PONTE é UMA PASSA; fr: 417310, 715301; sl: Ping-pongový stôl, Reklamný pútaÄ, Oprava nástenky, Knihobúdka, Tenisový kurt

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GT 5.2 Disk area	**Identify a disk, its perimeter, diameter or radius in order to calculate its area**
Curriculum item: level 6 (12 years old) Geometry Module 1º ESO *"To calculate the length of the circumference, the area of the circle."*	
	Data to be measured: The perimeter ($2\pi r$), diameter ($2r$) or radius (r) of the disk.
	Solution: Firstly, measure the radius, the diameter or the perimeter, and then use the formula $A=\pi r^2$ in the right units to calculate the area of the disk.
	Possible Hints: ● Use a unit pattern to measure the radius or diameter. ● Use a cord or a suitable tool to measure the diameter. ● Use the formula $A=\pi r^2$
	List of objects: circular stone (Area of the circle), manhole cover (Tapa alcantarilla), circular garden (Cí-rculo), round table (Picnic), millstone (Coroa circular)
List of referring numbers MCM tasks: de: Drehscheibe; en: Area of the circle; sp: Tapa alcantarilla ; pt: Coroa circular, Picnic, Círculo; fr: L'aire de l'interdiction;	

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GT 5.3 Cylinder area	**Determine the surface area of an object of/modelled by a cylindrical shape.**
Curriculum item: level 8 (14 years old) "To calculate areas and volumes of cylinders and apply them to solve contextualised problems."	
	Data to be measured: Perimeter P Height h The data should be measured several times at different locations to consider deviations from the chosen model. Do not forget to use the same units in both measurements. Solution: $A_C = P \cdot h$ Possible Hints: ● Which shape models the object? ● You can assume that it is a cylinder. Determine its surface. ● Area = perimeter multiplied by height.
List of objects Tree trunk (Stadtpanorama), Columns (Wo ist der Scheitel?, Coluna), Tubes (Tubos redondos pasar), Trash can (Limpieza en el Museo, Papelera, Del cilindro a la esfera), Balconies (Relva no miradouro), Posts (Les pots de peinture), Flowers bed (Nová výsadba), Billboard (Prehliadka plagátov), Benches (LaviČka)	
List of referring numbers MCM tasks de: Stadtpanorama; en: Wo ist der Scheitel?; sp: Tubos redondos pasar, Limpieza en el Museo, Papelera, Del cilindro a la esfera; pt: Coluna, Relva no miradouro; fr: Les pots de peinture; sl: Nová výsadba, Prehliadka plagátov, LaviČka	

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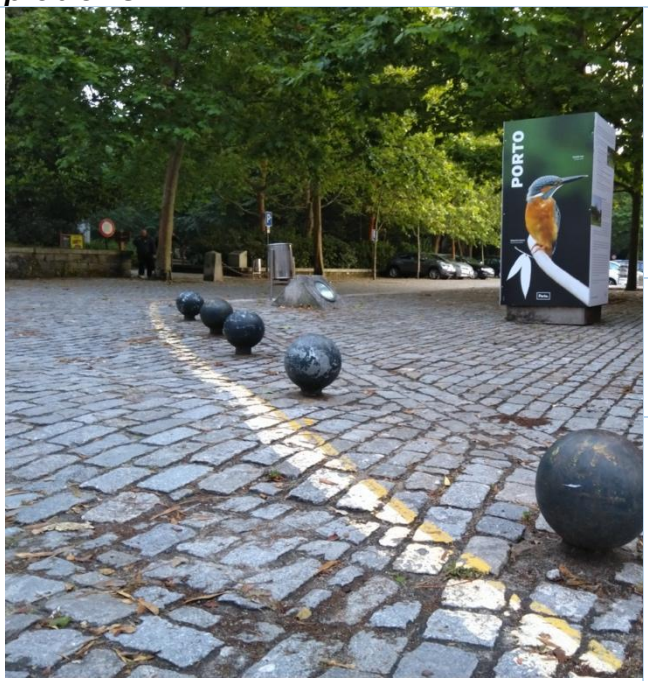
GT 5.4 Sphere area	**Determine the surface area of (an object modelled by) a sphere.**

Curriculum item: level 8 (14 -16 years old) "To calculate areas and volumes of spheres and apply them to solve contextualised problems."



Data to be measured:
Diameter D or Perimeter of a great circle P

The data should be measured several times at different parts to consider deviations from the chosen model.

Perimeter of a great circle P:
Be sure that the great circle is being measured.

Solution:
With the diameter estimation D, deduce its radius $R = D/2$.

With the Perimeter of a great circle, deduce its radius $R = P/(2\pi)$, $A_C = 4\pi R^2$

Possible Hints:

- Which shape models the object?
- You can assume that it is a sphere. Determine its surface area.
- Area = $4\pi R^2$
- You can get the radius by measuring the diameter.
- You can get the radius by measuring the perimeter (circumference) of a great circle.
- Circumference $P=2\pi R$

List of objects Buildings (Puerta Campo de la N) Paintings (La esfera) Sculptures (Oberfläche der Kugel,sphere) Signaling (Esferas) Reflector (Zrkadlo v reflektore)

List of referring numbers MCM tasks de: Oberfläche der Kugel; en: sphere sp: Puerta Campo de la N, La esfera; pt: Esferas; fr:Repeindre les boules; sl: Zrkadlo v reflektore

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GT 5.5 Cone area	**Determine the area of the curved surface of a Cone**

Curriculum item: level 9 "The pupil understands the cone elements: the* vertex lying directly above the centre of the base circle is apex, curved surface is formed by the line segments joining the vertex (apex) to the points of the base circle. *The area of a cone is the area of the curved surface and the number is a half of the product of the perimeter of the cone base circle by its slant height. To calculate areas and volumes of cones and apply them to solve contextualized problems."

Data to be measured: The perimeter of the base, the slant height from apex to perimeter, the height of the cone, the radius of the base circle.

Solution:
The lateral area A is the half of the product of the perimeter P of the cone base circle and the slant height S.

$$A = \frac{1}{2} P \times S$$

If the slant height cannot be directly measured, it might be computed from the height H of the cone and radius $R = P/2\pi$ of the base circle, using Pythagoras theorem :

$$S = \sqrt{H^2 + R^2}$$

Possible Hints:

- What is the shape of the object?
- Cone consists of the base circle, vertex (apex) and curved surface.
- Can you measure the height (slant height, surface, radius of the base circle)?
- Identify the base circle of the cone.
- Identify the apex of the cone.
- Use Pythagorean theorem for computing the slant height.
- Mind the units.
- Clarifying the shape, cone surface elements by photo or picture or drawing.

List of objects: sculpture (Moliendo, moliendo), glass pavilion (Kegel an der Messe), part of the pillar (La hauteur du cône de chantier), pine tree (Le cône Botanique), road safety cone (La surface du cône de chantier)

List of referring numbers MCM tasks de: Kegel an der Messe; en: ; sp:Moliendo, moliendo; pt:; fr: La surface du cône, Le cône Botanique,La hauteur du cône de chantier;

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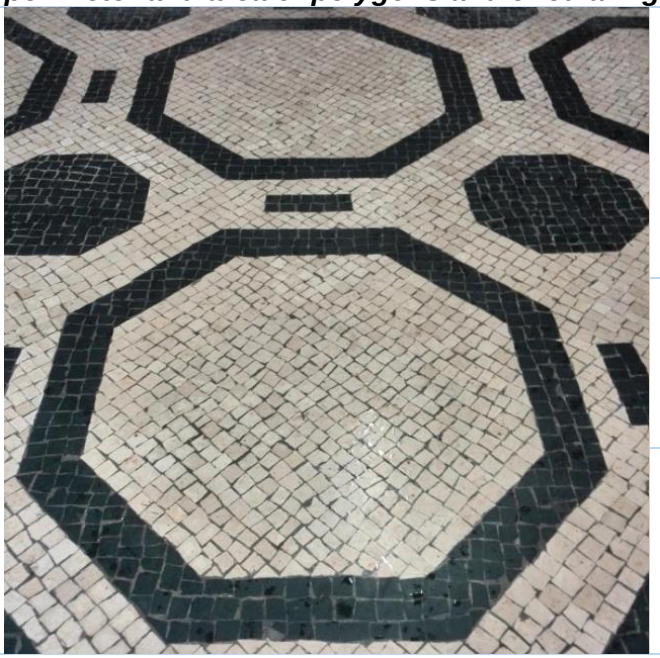
GT 5.6 Regular polygon area	**Find the area of a regular polyhedron.**

Curriculum item: level 8 "The pupil understands polygon as a plane figure bounded by some number of straight lines. The pupil identifies a regular polygon: all the sides have equal length and interior angles have equal size and the vertices lie on a circumcircle. To calculate the perimeter and area of polygons and circular figures in contextualised problems by applying appropriate formulas and techniques."



Data to be measured:
Perimeter of the regular polygon = sum of all sides lengths = one side length multiple by number of sides (or the number of vertices)
Apothem = regular polygon inscribed circle radius = perpendicular distance between the centre of the polygon and the appropriate polygon side = half of the perpendicular distance between two opposite sides
Regular (equilateral) polygon consists (or can be divided into) equilateral or isosceles triangles with one common vertex, which is the centre of the polygon.

Solution: In general: The area of the regular polygon is the perimeter multiplied by apothem divided by two.
The area of the regular polygon is the number of inscribed triangles multiplied by the area of the triangle.
Regular polygons: isosceles triangle, square, pentagon, hexagon, heptagon, octagon, enneagon or nonagon, decagon, hendecagon, dodecagon

Possible Hints:
- What is the shape of the object?
- Clarifying questions about the situation; e.g. How many elements are not visible? How can you decompose the surface into triangles?
- Clarifying shape and type of the polygon by photo or picture or drawing.

List of objects: flower frame (Blumenbeet), tiles in regular polygon shape (20, Amphithéatre du square Jean Cagocau), road sign (Hexagonal Stop), room bottom (Superficie de Kiosco), box cover (Studna), table (The hexagon) fountain (Custo de Restauro da)

List of referring numbers MCM tasks
de: Blumenbeet; en: 20, The hexagon, Hexagonal Stop; sp: Superficie de Kiosco; pt: Custo de Restauro da; fr: Amphithéatre du square Jean Cagocau4; sl: Studna

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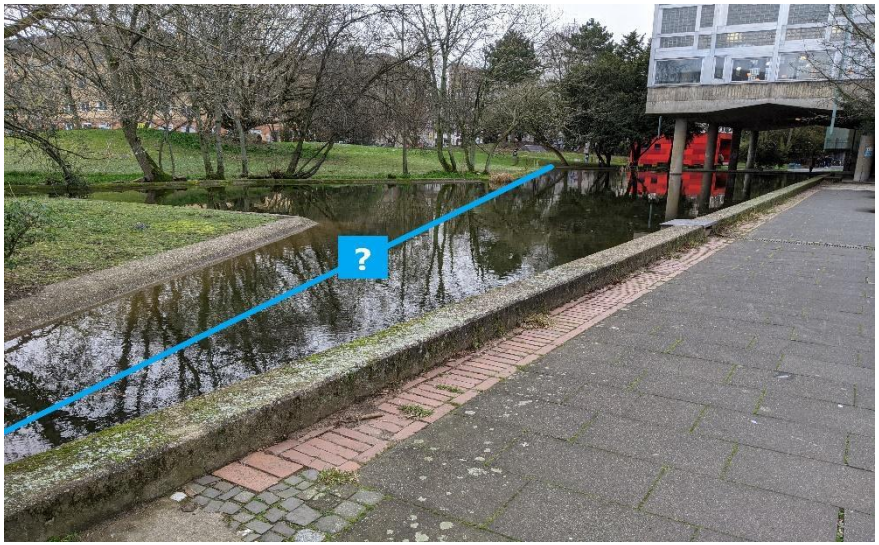
GT 5.7 Side of a rectangular triangle	**Determine a defined length using Pythagoras theorem**
Curriculum item : level 8 Pythagoras theorem	
 *If necessary, please highlight the straight-line segment, which length should be identified.*	<u>Data to be measured:</u> The lengths of two out of three sides of a rectangular triangle. <u>Solution:</u> Identify a rectangular triangle. Measure two of its sides. Calculate the length of the unknown side by using the Pythagoras theorem $a^2 + b^2 = c^2$ with c hypotenuse and a, b legs of a rectangular triangle. <u>Possible Hints:</u> ● Which geometrical shape can be used to identify the length of the unknown side? ● Identify a rectangular triangle, of which you can measure two sides. The length of the third side can be calculated of using the Pythagoras theorem. ● The Pythagoras theorem is $a^2 + b^2 = c^2$, whereas a, b are the legs of a rectangular triangle and c is its hypotenuse.
<u>List of objects</u> the diagonal of an object of rectangular shape, e.g. a rectangular fountain or flower bed. Fountains (Der herzögliche Brunnen), Lakes (Teichdiagonol), Dog waterer (Quanta água bebem os cães?)	
<u>List of referring numbers MCM tasks</u> de: Der herzögliche Brunnen, Teichdiagonol; en: Pyramid; sp:; pt: Quanta água bebem os cães?; fr: Énigme du marché;	

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GT 6.1 Cuboid volume	**Determine the volume (or the mass) of a (an object modelled by a) cuboid.**

Curriculum item: level 8 (15 years old) "To calculate areas and volumes of polyhedra and apply them to solve contextualised problems."

Data to be measured:
Length l Width w Height h
The data should be measured several times at different parts to consider deviations from the chosen model.
Do not forget to use the same units in all measures.

Determining the mass, one has to consider the density as well:
Concrete: 1,8-2,4 g/cm³
Granite: ρ= 2,6 g/cm³

Solution:

$$V = l \cdot w \cdot h$$

Determine the mass via multiplication with the density.

$$m=\rho \cdot V$$

Possible Hints:
- Which shape models the object?
- You can assume that it is a cuboid. Determine its volume.
- Volume = length multiplied with width multiplied with height.
- Measure the values in cm and multiply the volume with the density.

List of objects Stone (Stein, Stein, Stone) Swimming pool (Take a Deep in the P) Sculpture (base of a column Volumen de la base d, memorial stone Pedestal estatua de, Pamätná tabuľa) Benches (rocks Iglesia de San Nicol wood Holzklotz, Drevené hranoly) Buildings & rooms (phone cabin Cabinas Vintage) Street container (water Tanque, Encher o tanque sand Pieskovisko flower Flower pot) Packet (La avena colombiana)

List of referring numbers MCM tasks
de: Stein, Stein, Holzklotz; en: Stone, Take a Deep in the P, Flower pot; sp: Volumen de la base d,Pedestal estatua de, Iglesia de San Nicol, Cabinas Vintage; pt: Tanque, Encher o tanque; fr: La avena colombiana, Volume de la BU; sl: Drevené hranoly, Pamätná tabuľa, Pieskovisko

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GT 6.2 Cylinder volume	**Determine the volume or the mass of an object modelled by a cylinder.**
Curriculum item: level 9 (15 years old) "To calculate areas and volumes of cylinders and apply them to solve contextualised problems."	
	Data to be measured: Height h, Diameter d OR Perimeter/circumference P The data should be measured several times at different parts to consider deviations from the chosen model. Do not forget to use the same units in all measures. Determining the mass, one has to consider the density as well: Concrete: 1,8-2,4 g/cm³, Granite: 2,6 g/cm³, Soil: 1,2-1,5g/cm³
	Solution: With the diameter estimation d, deduce its radius $r = d/2$ OR With the circumference estimation P, deduce its radius $r = P/(2\pi)$. Determine de area of the base $A = \pi\, r^2$ and the volume: $V = l \cdot A$ Determine the mass via multiplication with the density: m=$\rho \cdot V$ It could also be interesting to calculate the mass of the pot. You have to build the difference between the Volume of the bigger cylinder and the Cylinder inside.
	Possible Hints: ● Which shape models the object? ● You can assume that it is a cylinder. Determine its volume. ● Volume = area multiplied with height. ● Area = $\pi\, r^2$ ● Get an estimation of the radius by measuring the diameter OR the circumference. ● Measure the values in cm and multiply the volume with the density.
	List of objects Fountain (Brunnenvolumen, Capacidad del estanq) Flower pot (Nicht (K)übel!, Pflanzenkübel) Column (Sé¤ule) Well (Wishing well) Tree trunk (Tronco de araucaria, TRONCO, Tetrov hlucháš) Pole (Lipstick de granito) Trash can (La papelera, Caixote do lixo, La poubelle de la li) Asphalt concrete (Asfaltobetónové polmesiačiky)
List of referring numbers MCM tasks de: Brunnenvolumen, Nicht (K)übel!, Sé¤ule, Pflanzenkübel; en: Wishing well; sp: Capacidad del estanq, Tronco de araucaria, Lipstick de granito, La papelera; pt: Caixote do lixo, TRONCO; fr: La poubelle de la li; sl: Tetrov hlucháš, Asfaltobetónové polmesiačiky	

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GT 6.3 Sphere volume	**Determine the volume or the mass of an object modelled by a sphere.**

Curriculum item: level 9 (15 years old students) "To calculate areas and volumes of spheres and apply them to solve contextualised problems"

Data to be measured:
Diameter D or Circumference/perimeter P of a great circle
The data should be measured several times at different parts to consider deviations from the chosen model.
Determining the mass, one has to consider the density as well:
Concrete: 1,8-2,4 g/cm³
Granite: 2,6 g/cm³

Solution:
With the diameter estimation D, deduce its radius $r = D/2$ OR
With the circumference estimation of a great circle P, deduce its radius $r = P/(2\pi)$ then$V = \frac{4}{3}\pi \cdot r^3$
Determine the mass via multiplication with the density.

Possible Hints:

- Which shape models the object?
- You can assume that it is a sphere. Determine its volume.
- Volume = $\frac{4}{3}\pi \cdot r^3$.
- Get an estimation of the radius by measuring the diameter OR the circumference of a great circle.
- Measure the values in cm and multiply the volume with the density.

List of objects Stone (Kugel am Kriegssteig, Weight of the sphere, Fuente junto al Acue, Esfera) Flower pot (MACETEROS SEMIESFÉR)

List of referring numbers MCM tasks de: Kugel am Kriegssteig; en: Weight of the sphere; sp: MACETEROS SEMIESFÉR, Fuente junto al Acue; pt: Esfera; fr: La boule de l'espace;

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GT 6.4 Cone volume	**Determine the mass/volume of a (possibly truncated) cone.**
Curriculum item: level 9 "The pupil understands the cone elements: apex, base circle, height. The volume of a cone is a one third of the product of the area of the cone base circle and the cone height. To calculate areas and volumes of cones and apply them to solve contextualized problems"	
	Data to be measured: The radius or diameter of the cone base circle, the height of the cone: from the apex to the centre of the base circle; the slant height of the cone: from apex to the perimeter of the cone. In the case of truncated cones, measure the radius or diameter of the top and bottom circles and the height of this truncated cone.
	Solution: The volume V of a cone is one third of the product of the cone base circle area A (calculated from it radius R by $A = \pi R^2$) and the cone height H. The cone height H might be computed from the slant height S of the cone and radius R of the base circle, using Pythagoras Theorem: $H = \sqrt{S^2 - R^2}$ In the case of a truncated cone, may use the formula V= 1/3*(R^2+R*r+r^2)*h*pi. where the small letters corresponds to the new lengths : r is for the radius of the small circle and h is the height of the truncated cone Determine the mass via multiplication with the density.
	Possible Hints: ● What is the shape of the object? A Cone consists of the base circle, vertex (apex) and curved surface. ● Can you measure the height (slant height, surface, radius of the base circle)? ● Identify the base circle of the cone. Identify the apex of the cone. ● Use Pythagorean theorem for computing the height. ● The volume is a third of the product of the base area by the height. Mind the units. ● Clarifying the shape, cone surface elements by photo or picture or drawing.
List of objects: sculpture (Piedra de molino), glass pavilion, part of the pillar, pine tree, road safety cone, concrete cone (Mass of a Pillar, Concrete Cone), flowerpot (Dates, Le volume du pot)	
List of referring numbers MCM tasks de: Dates, en: Mass of a Pillar, Concrete Cone, sp: Piedra de molino, pt:, fr: Le volume du pot	

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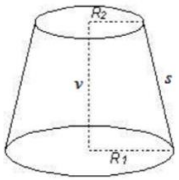
GT 6.5 Truncated cone volume	**Determine the mass/volume of a truncated cone.**

Curriculum item: level 10 "The pupil understands the cone elements: apex, base circle, height. To calculate areas and volumes of cone trunks and apply them to solve contextualized problems."

Data to be measured: The radius or diameter of the cone base and top circles, the height of the truncated cone: from the centre of the smaller (top) circle to the centre of the base circle; the slant height of the truncated cone: from a perimeter of the top circle to the perimeter of the base circle.

Solution:



$$V = \frac{\pi v}{3} \cdot (r_1^2 + r_1 \cdot r_2 + r_2^2)$$

The truncated cone height might be computed from the slant height of the truncated cone and radius of the base circle, using Pythagoras Theorem.

Possible Hints:

- What is the shape of the object?
- Truncated cone consists of the base circle, top circle and curved surface.
- Can you measure the height (slant height, surface, radius of the circles)?
- Identify the base circle (the top circle) of the cone
- Use the Pythagorean theorem for computing the height.
- Mind the units.
- Clarifying the shape, cone surface elements by photo or picture or drawing.

List of objects: sculpture, glass pavilion, part of the pillar, pine tree, road safety cone (Concrete cone, La hauteur des cônes), flower pot (Le volume du pot)

List of referring numbers MCM tasks
de: Betonkegel (DE); en: Concrete cone; sp: ; pt:; fr: La hauteur des cônes, Le volume du pot

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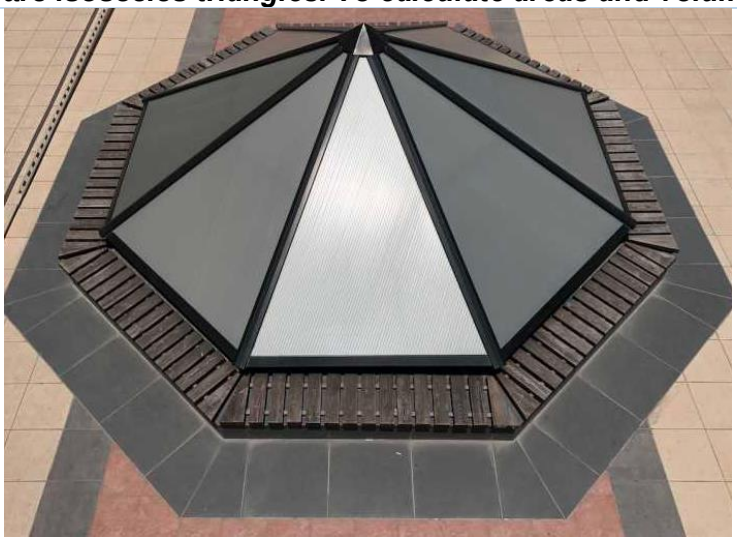
GT 6.6 Pyramid volume	**Determine the mass/volume of a (possibly truncated) pyramid.**

Curriculum item: level 10 "The pupil understands the pyramid elements: base polygon, height, apex, slant height. The* vertex lying directly above the centre of the base polygon is the apex. *The volume of a pyramid is a third of the product of the area of the pyramid base polygon and the pyramid height. A right-regular polygon is one in which the base is a regular polygon and the remaining faces are isosceles triangles. To calculate areas and volumes of pyramids and apply them to solve contextualized problems."



Data to be measured: The area of the base. The height of the pyramid. The slant height of the pyramid.

Solution: Volume of the pyramid is a third of the product of the area of the base of the pyramid and its height. The height of the pyramid might be computed using the slant height and appropriate element of the base using Pythagorean Theorem.

Possible Hints:

- What is the shape of the object?
- Pyramid consists of a base polygon and triangular faces.
- Can you measure the height (slant height, surface, side of the base)?
- Identify the base polygon of the pyramid.
- Use the Pythagorean theorem for computing the height.
- Mind the units.
- Clarifying the shape, pyramid surface elements by photo or picture or drawing.

List of objects: sculpture (Pyramidenbaum, Volumen der Pyramide), glass pavilion (Stahlpavillon), part of the pillar (Pyramide), roof (Rotes Pyramidendach), crystal, cover

List of referring numbers MCM tasks
de: Pyramidenbaum, Pyramide, Stahlpavillon, Volumen der Pyramide, Rotes Pyramidendach, Pyramide; en: Pyramid height, sp: , pt:, fr:

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GT 7.1 Cyclic Duration	**Time the duration of a regular event**
	Data to be measured: Time a regular event.
	Solution: Measure the duration of this regular event. If the number of the event's repetitions per given time period should be identified, calculate the number of repetitions per minute, hour, day or year.
	Possible Hints: ● Measure the duration of this event. ● How often is this event repeated per minute? ● How often is this event repeated per hour (per day; per year)?
List of objects A regular event: - an object of uniform velocity, e.g. elevator, mechanical stairs - a periodically repeated moving object, e.g. a rotating advertising pillar Moving sculpture (Hammering Man, Hammering Man(eng)) Moving column (Werbedrehwurm, Drehende Litfaésäu) Staircase (Rolltreppe an der Ha) Round advertising (Ich dreh ab!, Dauerwerbesendung) Lift (Der Lift) Feet washer (Lavas tus pies) Fountain (Encher piscina coa auga das Burgas, El agua del riñón) flashing light (Walk the line) sliding door (Time to close)	
List of referring numbers MCM tasks de:Hammering Man, Werbedrehwurm, Rolltreppe an der Ha, Drehende Litfaßsäule,Ich dreh ab!, Der Lift, Dauerwerbesendung; en: Hammering Man, Time to close; sp: Lavas tus pies, Encher piscina coa auga das Burgas, El agua del riñón; fr: Walk the line,	

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GT 7.2 Duration	**Determine the duration of a not cyclic event.**
 *The photo has to clarify the object or event of the task. However, it must not show the relevant dates.*	Data to be measured: Identify differences in time between the beginning and end moment of one event, based on text information, actual measurement with a chronometer. Solution: Identify the duration of this event. Therefore, the difference between the beginning and end of the event has to be calculated. Possible Hints: ● Read the information board. Look for the relevant data. ● Calculate the duration of this event. ● To calculate the duration of the event, you have to subtract the end moment from the start moment. ● Centuries, years, months, days, hours, minutes, seconds, beware of the units.
List of objects Ages (ZONTA-Club, Age of Miguel de Cervantes Saavedra, Dates, Bailaor Mario Maya, Antiguedad, Historia de Coyanza, Idade) Historical events (Geburtstag, Dichter & Denker) Duration of construction (Le monter c'est long, Obras de Eduardo Bla)	
List of referring numbers MCM tasks de: ZONTA-Club, Geburtstag, Dichter & Denker; en: Age of Miguel de Cervantes Saavedra, Dates; sp: Bailaor Mario Maya, Obras de Eduardo Bla, Antiguedad, Historia de Coyanza; pt: Idade; fr: Le monter c'est long;	

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GT 7.3 Flow	**Determine the flow as the ratio of a quantity per corresponding time**

Curriculum item: (in Spain) level 7 (13 years old). *"To identify and discriminate relationships of numerical proportionality and use them to solve problems in daily situations." "Reason and proportion. Directly and inversely proportional magnitudes. Constant of proportionality".*

Data to be measured: A quantity in a given time, for example, litres per minute. Understand the synchronization of two measures. Precision in synchronized measure of time and quantity. Volume of the recipient given, and time needed to fill it. Determine the right units.

Solution: The ratio between two elements. Or given the flow and one element, compute the other one.

Possible Hints:

- Determine the two elements involved in the flow.
- Calculate the ratio between the two corresponding elements.
- Choose a recipient and units to determine the volume, it can be a graduated recipient or a recipient which you can easily determine the volume.
- Volume formula.

List of objects: watercourse (Wasserfluss der Pader), coffee machine (Débit volumique d'une machine à café), fountain (Der Fluss, ¡La fuente se desborda!, Fluxo, "El agua" de Eugenio), tank (Wasser), graffitis (Graffitis), bin (Volumen de la papele)

List of referring numbers MCM tasks: de: Der Fluss, Wasser, Wasserfluss der Pader; en: sp: Volumen de la papele, ¡La fuente se desborda!; pt: Fluxo; fr: Débit volumique d'une machine à café;

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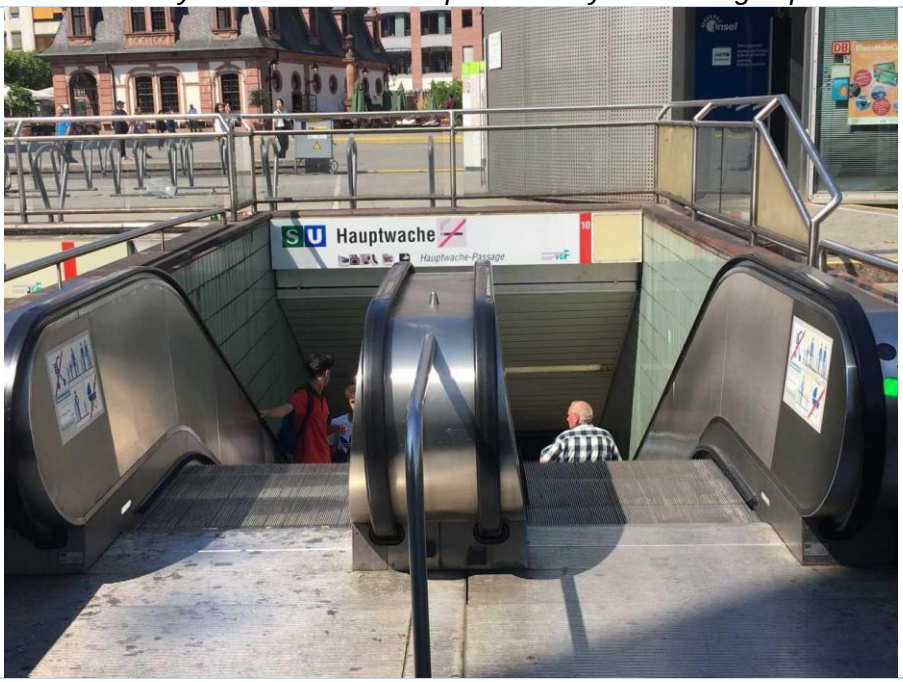
GT 7.4 Average velocity	**Determine the velocity as the ratio of distance per time**

Curriculum item: level 7 (13 years old) *"To identify and discriminate relationships of numerical proportionality and use them to solve problems in daily situations.""Reason and proportion. Directly and inversely proportional magnitudes. Constant of proportionality". Currículo ESO Physics and Chemistry – To determine experimentally the average speed of a body. Average speed.*



Data to be measured: The time for a corresponding distance. Precision in synchronized measure of time and distance. Determine the right units.

Solution: The velocity as the ratio between distance and time in the right units. It requires precision in synchronized measure of time and distance.

Possible Hints:

- Determine a distance and the time required.
- Calculate the ratio between distance and time.
- Sometimes it will be necessary to calculate the average among several estimations.
- Calculate the average time in an escalator from to bottom and from top to bottom.
- Measure the length of the entire escalator or just one step and calculate.
- Number of decimal places to round the result.

List of objects: escalators (Rolltreppe Westend, Rolltreppe, Geschwindigkeit, EXPRESSWAY), tramway, barber pole (Poste de barbero), fountain (Repuxos), lift

List of referring numbers MCM tasks: de: Rolltreppe, Rolltreppe Westend, Geschwindigkeit der; en: speed; sp:Poste de barbero, EXPRESSWAY; pt: Repuxos; fr: Vitesse du Tram;

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GT 7.5 Maximum velocity	**Determine the velocity as a function of time and the constant acceleration as a piecewise linear model.**
	Data to be measured: Model speed as a function of time and find its maximum. Be careful that the task doesn't involve any dangerous strategies to evaluate the maximum speed. Solution: To calculate the maximum speed constrain the model by the average speed. Possible Hints: ● Speed is distance divided by time ● Estimate the portion where you think the speed is maximum and estimate a distance between two markers and coordinate to time. ● Make several estimations
List of objects: tramway (Vitesse du tram)	
List of referring numbers MCM tasks: fr: Vitesse du tram	

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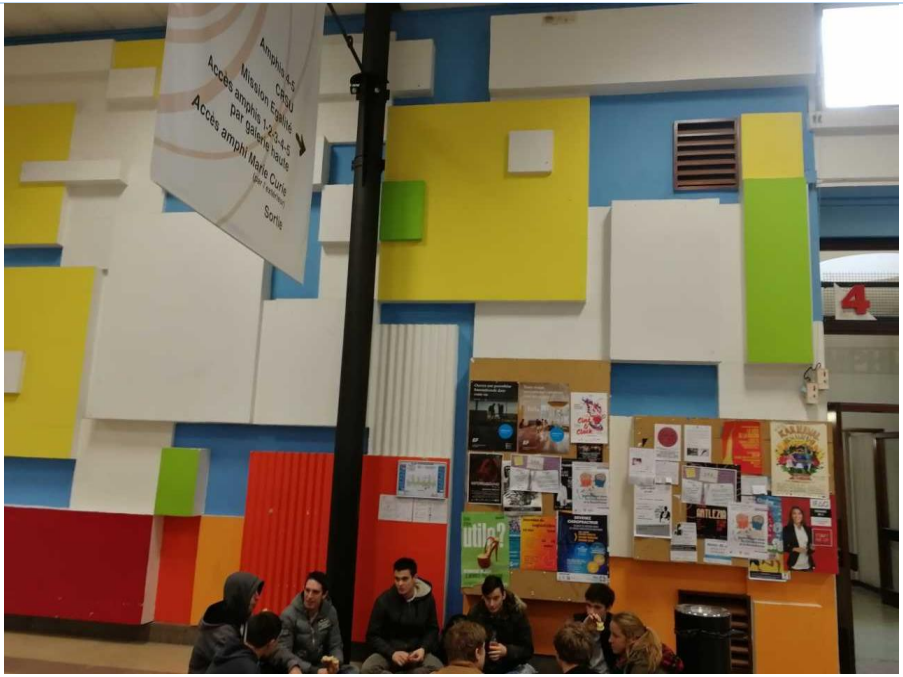
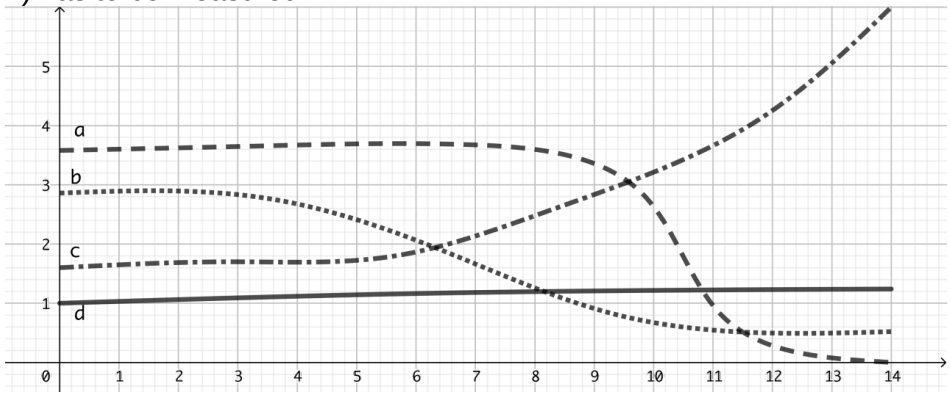
GT 7.6 Function	**Model a phenomenon with a function.**
 *The photo has to clarify the object or event of the task. However, it must not show the relevant details. The framework of reference should be made clear. In this example, which side of the corridor is the origin of the frame? The obvious choice where x grows going from the left to the right should be explicit.*	<u>Data to be measured:</u> Identify evolution in space of a certain quality and model it with a function. In the given example, proportions of colours vary along this corridor, the evolution of white/yellow/green and red colours can be associated with a multiple choice a/b/c/d in a given reference. Focussing on particular aspects (increasing/decreasing/extremum/periodicity) is interesting. Another example: find a periodic pattern and model it with a sin wave of which the wavelength (in m) or spatial frequency (in rad/m^-1) has to be measured.  <u>Solution:</u> Identify the evolution of the quality and notice the characteristics of the evolution, increasing/decreasing/ extrema in order to match it with a given pattern.

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	Possible Hints: • Identify the quality to trace, try to draw its evolution. Notice the salient features of this evolution, is it increasing, decreasing, does it have a maximum, is it null, positive, negative, where are these points? • Describing a function by its salient points through a table of variations is usually enough to identify it. • You might have to measure some features in order to identify the function.

List of objects Anything that features a quantity that varies continuously, or shapes that can be modeled with functions, bridges, lines, rooftops, slides, gates…

- Linear function : ramp (Lineare Funktion, Funktion im LPM), sliding chute (Parabelrutsche)
- Quadratic function : fence (Drevené hranoly), organ (The parabola)
- Polynom : curved roof (Tejado polinómico - find the minimal degree)
- Sinusoïd : oscillating fence (Le train sinueux - find the spatial frequency)

List of referring numbers MCM tasks de: Lineare Funktion, Funktion im LPM, Drevené hranoly, Parabelrutsche; en: The parabola; sp: Tejado polinómico; pt: ; fr: Le train sinueux; de: Denkmal Erlangen, Heidis Unterarm, Baumschutz, Quakis Algenplantage, Sinustor und Co(s), Brunnen, Springbrunnen, Türbogen, Züge, Parabelrutsche, Weltkarte, Astparabel, Parabelhandlauf, Parabeltor, Rutschbahn, Die Parabel im Zaun, Parabelrutsche, Wo ist der Scheitel?, Bachparabel, Tor, Parabel am Tür, Raucherplatz, Wäscheleine, Hängebrücke; en: Parabola, The parabola organ, Hurdle Jump, The speed; sp: Funciones, Tejado polinómico, Chorros de agua en l, Las palmeras, Función Lineal, Circunferencias, Catenaria o parábola?; pt: fr: le point selle, 949, Courbes des balcons, Le train sinueux, La fonction des volutes, La longueur d'onde; sl: Mriežka - súradnic; it: Le parabole del Giar, Le parabole del giar, La traiettoria, Acqua a catinelle

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Generic Tasks in the Context of MathCityMap@home

Within the context of distance education during the Corona pandemic, new challenges arise for teachers and students. The interplay of the synchronous and asynchronous features of the MathCityMap system is a promising approach for meeting the challenges of distance learning. The idea has been taken up as the basis for the MathCityMap@home concept that has been developed during the Corona pandemic for supporting mathematics education in distance learning.

MathCityMap@home still uses the original concept and the two components of MathCityMap. As in the out-of-school context, teachers create tasks and math trails in the sense of mathematical learning paths for their students in the web portal. The students download this path to their smartphone and solve the tasks using the hints and automatic solution checking. In addition to the previously entered hints and sample solutions, the Digital Classroom in MathCityMap@home offers teachers the possibility to interact synchronously with the students.

In contrast to the original concept, however, the tasks of MathCityMap@home are set to be solved not only on site but also at home. Numerous ideas from the generic tasks, e.g. linear functions or combinatorics, are particularly suitable. Suitable tasks for MathCityMap@home are already existing MathCityMap tasks, in which real measured values are added to the task text or task picture. It is also possible to enter classic textbook tasks or problems into the system.

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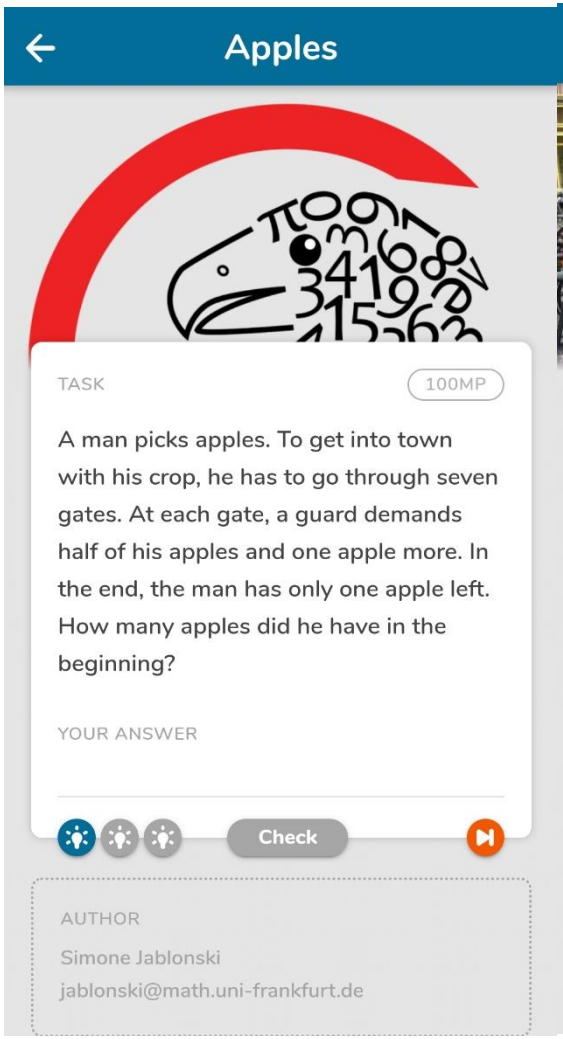
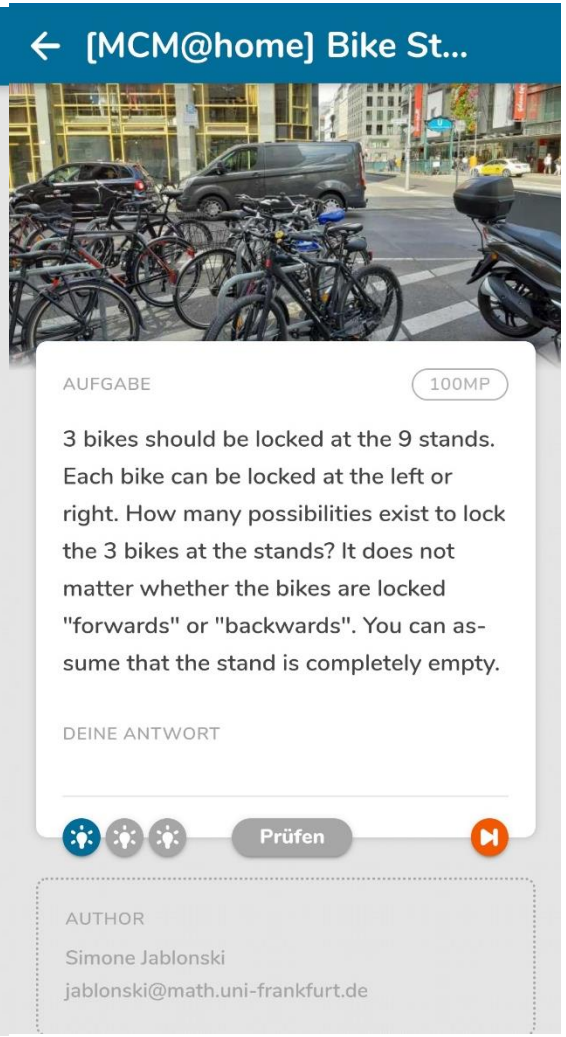
Generic MathCityMap@home Tasks: Combinatorics





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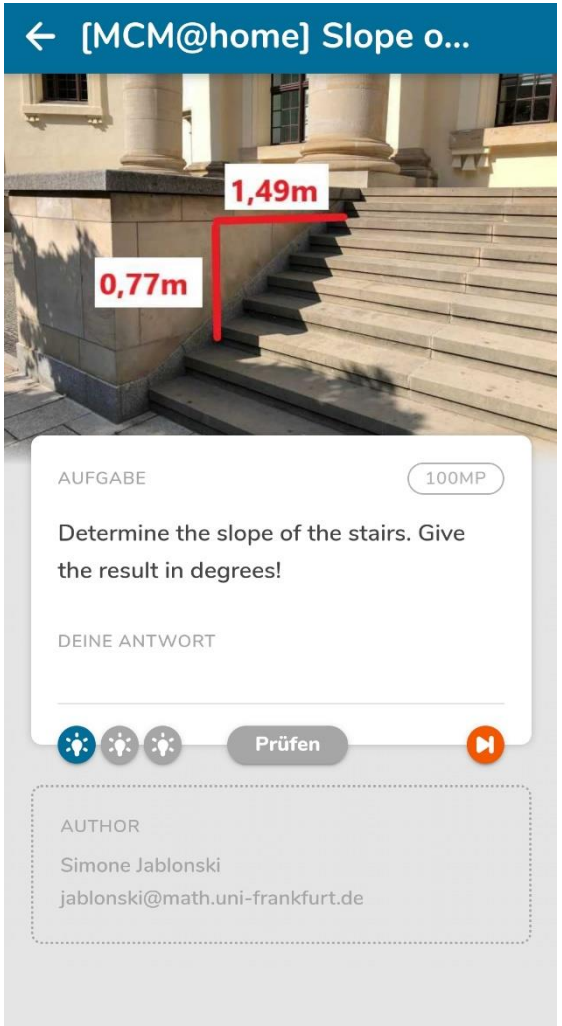
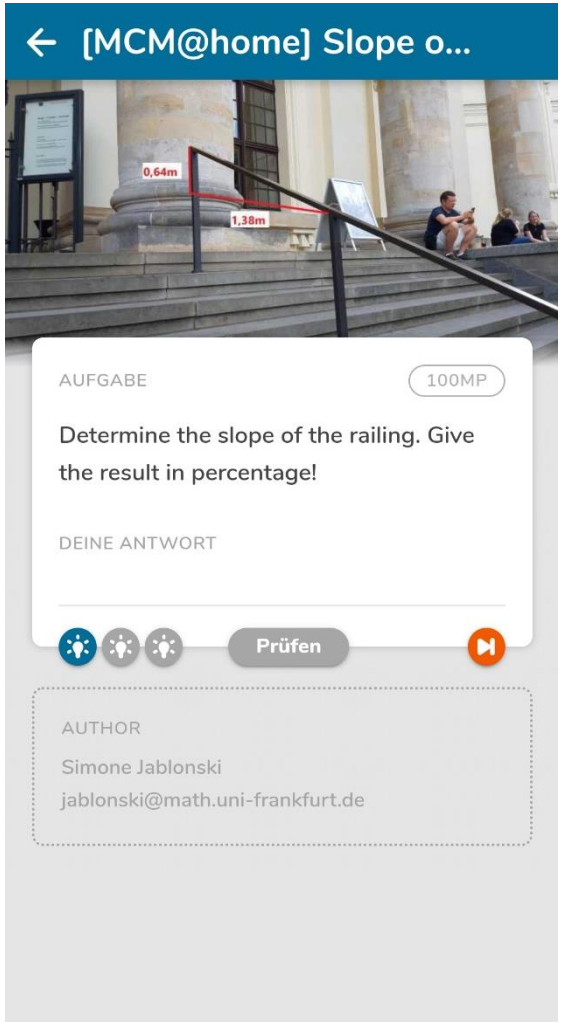
Generic MathCityMap@home Tasks: Slope





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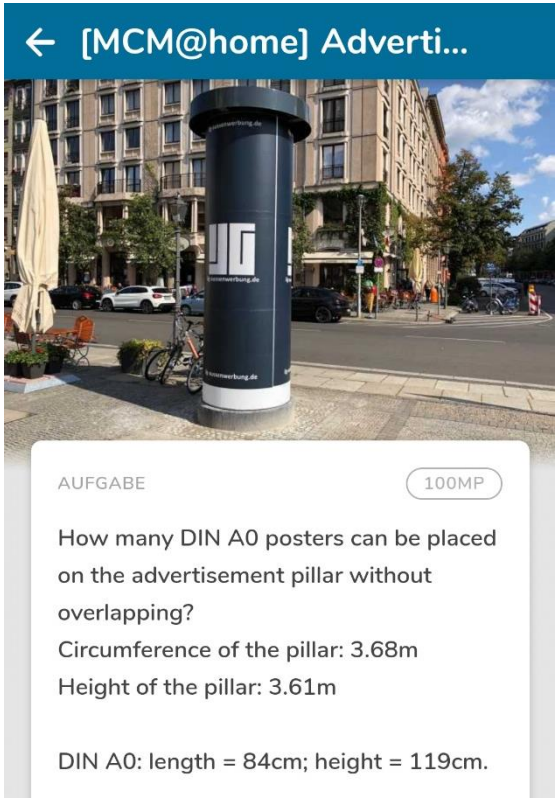
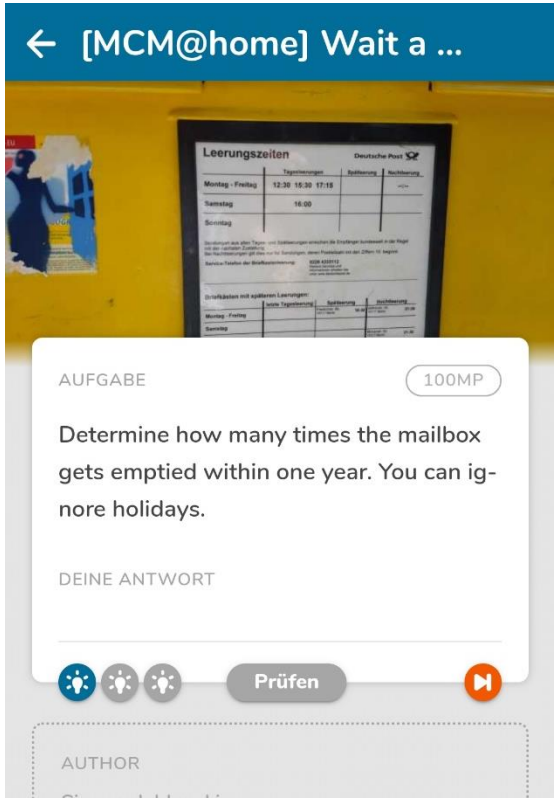
Generic MathCityMap@home Tasks: Quantities





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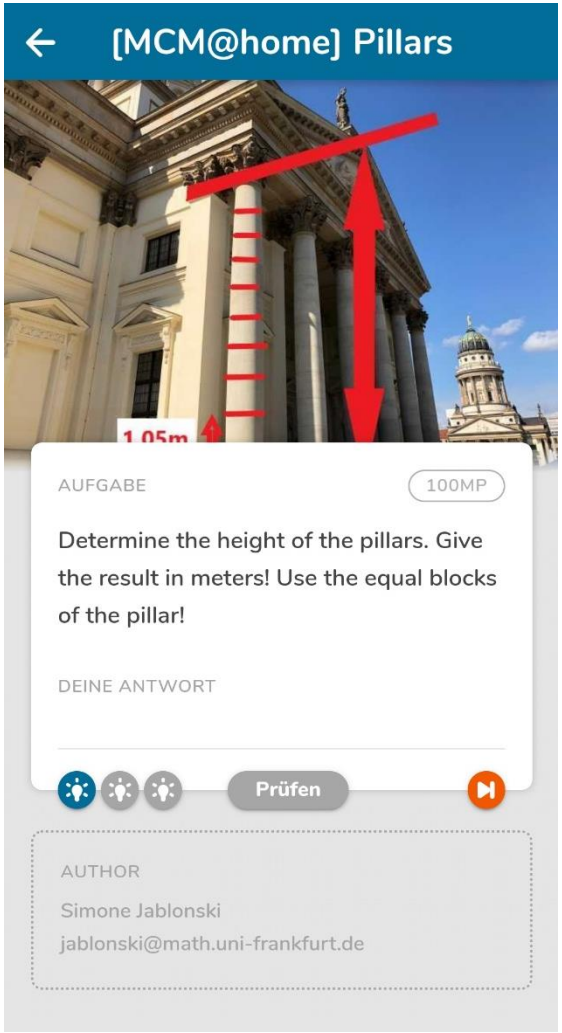
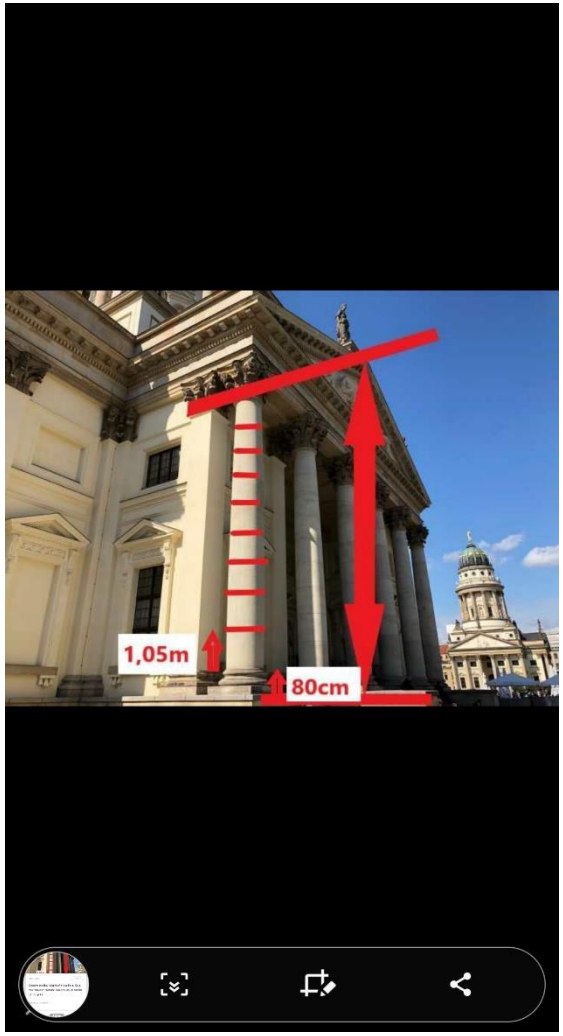
Generic MathCityMap@home Tasks: Height





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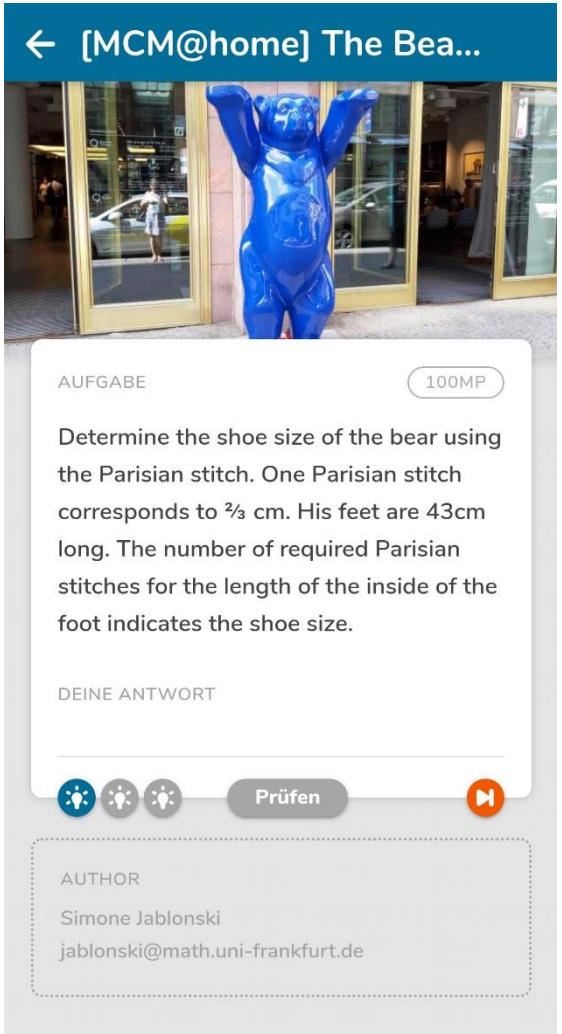
Generic MathCityMap@home Tasks: Proportion



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