Output 8: Validation and Research

During the development of math trails with new technologies and the corresponding material, we constantly worked on the project's validation and conducted research on different theoretical frameworks and aims.

Validation

Quality Criteria and Review System

MathCityMap is based on an app and a web portal in which every registered user is allowed to create and publish their own tasks. Through a constantly growing community and the aspiration for a certain quality of the published material, the system offers a multistep review process and several criteria for published content. The tasks and trails were continuously evaluated by experienced project MoMaTrE community researchers. In August 2020, the number of reviewers is 27, whereby the communication takes place in 12 languages: English, German, French, Spanish, Portugal, Slovak, Italian, Estonian, Turkish, Greek, Bahasa Indonesian and Chinese.

In the review process, task and trail authors are instructed in detail how to modify their items for publication on the project portal in case they are not accepted immediately. This open professional validation process resulted in 9.205 reviews, from which 69% were directly approved and 31% declined to be reworked (data in August 22, 2020). The ratio between approved and declined tasks and trails evidently shows the high quality of the MoMaTrE material.

This idea leads to the necessity of a catalogue of criteria which enables authors to comprehend the feedback and reviewers to give a transparent feedback on submitted tasks. With the help of relevant literature and through years of experience, the following catalogue has been developed.

1. Uniqueness. To make clear which object is meant, every task should provide a picture that helps to identify the object of the task and what the task is about.
2. Attendance. The tasks should be authentic in the sense of leaving the educational context and having a certification (Vos, 2013). Therefore, it is indispensable that a task can only be solved at the location of the object (Ludwig, Jesberg & Weiß, 2013). This also means that the picture and description of a task should never be enough to expose the solution.
3. Activity. Especially the physical activity has a positive effect on the cognitive learning, which is an important basis for the idea of embodied mathematics in the sense that mathematic concepts can only be fully comprehended through an active experience (e.g. Tall, 2013).

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The task solver should therefore become active and do something in order to solve the task, e.g. measure and count.

4. Multiple solutions. Authentic and modelling tasks are characterized by the fact that they are solvable in different ways through the choice of a mathematical model. The task should therefore be solvable in various ways.
5. Reality. A further important characteristic in this context is the connection of mathematics and emotions, interest and relevance for the students – aspects that significantly correlate with performance (e.g. Tulis, 2010). The task should have meaningful relevance and not appear too artificial.
6. Hints. As Jesberg & Ludwig (2012) summarize, several studies come to the conclusion that stepped aids have a positive impact on learning performance, experience and communication (Jesberg & Ludwig, 2012). Therefore, every task should provide at least one hint in terms of solving the task.
7. School math and tags. The task should feature a connection to school math. Therefore, one can use tags with relevant key words and assign each task a target grade.
8. Solution formats. MCM provides different solution formats: interval, exact value and multiple choice. The solution should be representable in one of these formats. Especially for modelling tasks, the interval seems very relevant as it enables to refrain from minor deviations in the solution, as through measuring differences or different mathematical models. In this format, one defines a green interval for good solutions, and an orange interval for not good, but acceptable solutions. All solution values that do not fit into these intervals receive the feedback wrong and ask the player to retry.
9. Tools. The task should be solved without special and extraordinary tools apart from calculator, measuring tape etc.
10. Sample solution. One should provide a sample solution including measured data (only visible in the portal and in the solution PDF) for teachers in order to talk about the tasks in the following lessons and analyze typical errors.

Quantitative Statistics on MathCityMap:

The Graph 1 documents the development of tasks and users since the portal MathCityMap has been launched. In addition, Table 1 gives an overview on the annual growth of tasks and users during the MoMaTrE project since its start in September 2017.

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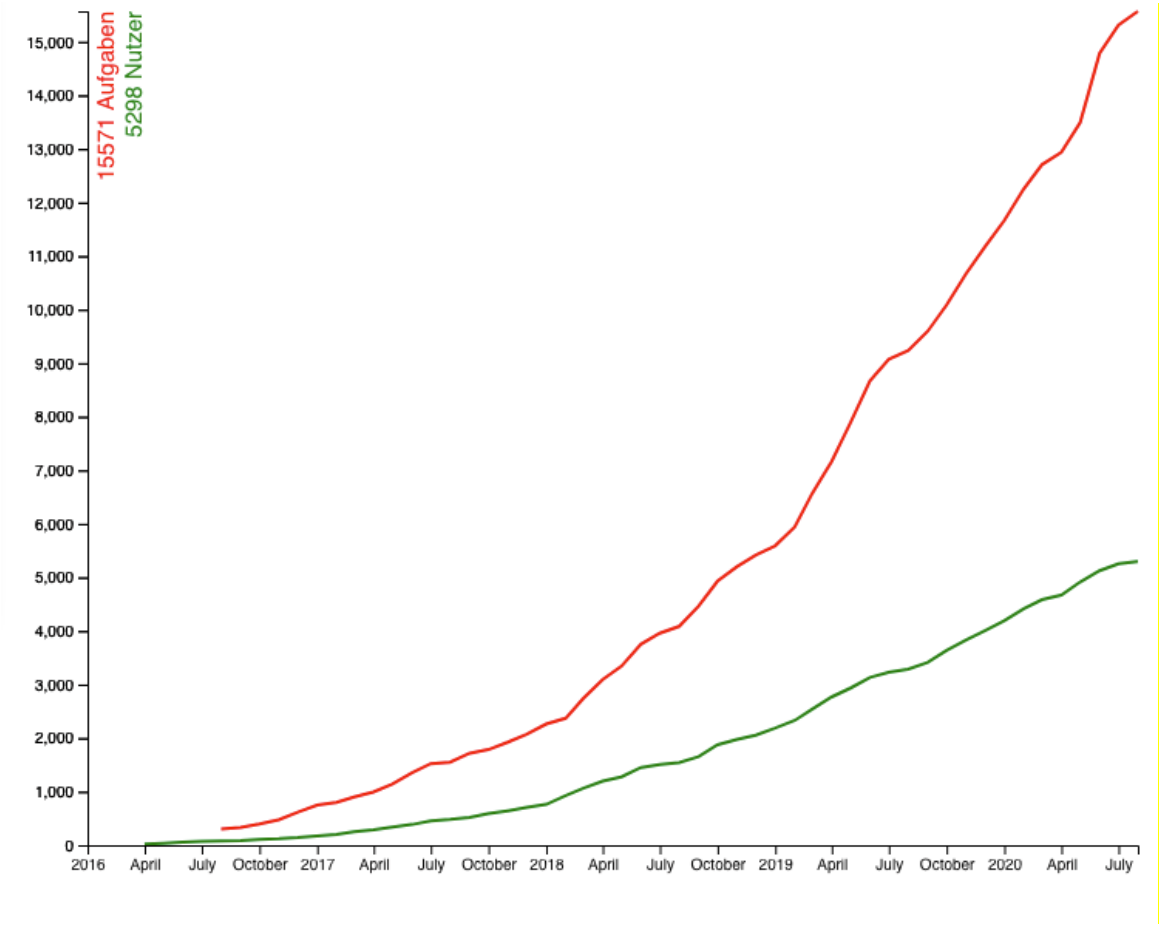

Graph 1. Development of Users and Tasks since the launch of MCM Portal (downloaded in August 22, 2020)

Table 1. Development of the MathCityMap Tasks and Users with the annual Growth in Percentage

Category	September 2017	September 2018	September 2019	August 2020
Tasks	1.578	4.095 (+160%)	9.258 (+126%)	15.569 (+68%)
Users	473	1.545 (+327%)	3.292 (+113%)	5.299 (+61%)

Summarizing the statistics on the basis of different indicators leads to the following quantitative project key performance indicators:

- The number of mobile tasks (15.500) and trails (2.500) produced by the teachers and the partners
- The number of Generic Tasks created by the consortium (40) and number of created wizard tasks (4.330)
- The number of conducted teacher trainings (multiplier events and additional small teacher trainings: 50)
- The number of downloads of the mobile application (Android: 14.700, iOS: 8.700)
- The number of registered users in the web portal (5.300)

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On average, a MathCityMap user has created three tasks. Nevertheless, we know that 20% have more than ten tasks which can be regarded as the active MathCityMap authorship community.

We assume that the strong growth of tasks is the evidence of the strong contemporary need in education (not only in mathematics): using digital technologies in education, connect these new technologies with real world, real objects and real life of pupils or users. Further, we see a validation of our dissemination strategies and especially the community website. Nevertheless, we would have expected an exponential growth, but due to the Corona pandemic, there was a stop in this increase. In all participating countries, the students were locked down which resulted in teachers being busy with creating online courses and new learning formats. Nevertheless, the global situation with COVID-19 pandemic since March 2020 did not stop the project activities, even though the schools were closed and the outdoor activities were restricted. The MoMaTrE leaders invented MathCityMap@home tasks and trails and project authors and contributors adapted their experience to this innovative approach. Many teachers and students in project consortium institutions but also in other schools and countries used MathCityMap@home trails as very welcome mathematics activity for pupils staying home during COVID-19 schools close. In total, 48 MathCityMap@home trails were created in German, Slovakian, Portuguese, French and Spanish. All in all, they were downloaded 1.250 times. Some of these special trails were composed as mathematics tests for certain school grades or of special mathematics topic knowledge or mathematics competence testing.

Evaluations

The main validation activities have been the evaluations of the activities and products. The evaluations of the Intensive Study Programme, Online Teacher Training and the Long-Term Curriculum are included in the Attachment Evaluation and Monitoring, as well as the User Survey among all MathCityMap users. All of them show that the participants did not encounter outdoor mathematics very often before, as well as a growth in the intention to use outdoor mathematics more often after participating in one of those activities.

Research

Many results to math trails related research questions have been published and are available for a broader auditorium in the open-access articles on the project website. There are 56 articles and conferences contributions in several languages available. More than 5.000 downloads of the publicly available items are the evidence of remarkable interest in these results of research based on the MoMaTrE project material and activities. The main idea for sharing not only high quality material, but also well described and validated methods of: how to use material, how to implement methods; instructions: how to prepare an own trail, how to organize a digital classroom and how to evaluate pupils' (participants) results, has been fully fulfilled. The broader auditorium is invited also to study the accessible data so any other scientists can use any project result for further analysis and research.

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The main research activities are:

(1) two quantitative studies on the learning and motivational aspects of math trails and in the second part math trails with different gamification elements,

(2) the publication of the proceedings of the ROSETA conference (see ME 2; https://www.wtm-verlag.de/DOI-Deposit/978-3-95987-144-0/978-3-95987-144-0-Book.pdf),

(3) several publications on math trails in different contexts, (e.g. modelling, giftedness and measuring),

(4) several publications on the project's results and activities (e.g. the ISP, the generic tasks and the features of the app).

Published Books

Open-Access

M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020. Münster: WTM. https://doi.org/10.37626/GA9783959871440.0 ISBN978-3-95987-144-0

Others

Zender, J. (2019). *Mathtrails in der Sekundarstufe I. Der Einsatz von MathCityMap bei Zylinderproblemen in der neunten Klasse*. Münster: WTM.

Published Articles:

Open-Access

Barlovits, S. & Ludwig, M. (2020). Mobile-Supported Outdoor Learning in Math Class: Draft of an Efficacy Study about the MathCityMap App. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020 (pp. 55-62). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.07

Barlovits, S., Jablonski, S., Milicic, G., Ludwig, M. (2020). MathCityMap@home: Digitale Lernpfade mit gestuften Hinweisen und synchroner Schüler-Lehrer-Interaktion. *Mitteilungen der Gesellschaft für Didaktik der Mathematik* , (109) S. 39-43.

Baumann-Wehner, M., Gurjanow, I., Milicic, G. & Ludwig, M. (2020). Analysis of Student-Teacher Chat Communication during Outdoor Learning within the MCM Digital Classroom. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), Research on Outdoor STEM Education in the digiTal Age (pp. 63-70). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.08

Blanco, J.C., Lázaro, C., Recio, T. (2019). El proyecto MoMaTrE: Paseos matemáticos con móvil por Europa. Boletín informativo de la SMPC , núm. 19, pp. 6-7. https://drive.google.com/file/d/1YC8wcVAaxelWOhW5HNE71eviB2kXDfwO/view

Blanco J.C., Lázaro C., Recio T. (2020). ErasmusDays 2019: Una ruta matemática por Santander con MathCityMap y MoMaTrE. Boletín informativo de la SMPC , núm. 20, pp. 6-8. https://drive.google.com/file/d/1gofYJf09Ja1_m3bxr2DdMspAM5TuyDFX/view

Caldeira, A. & Ceretkova, S. (2020). MathCityMap Generic Tasks in Portugal and in Slovakia. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020 (pp. 201-204). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0

Caldeira, A. Moura, A. & Mercat, C., (2019). Big events in Mathematics using math trails. In: U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education, (pp. 2804-2805). Utrecht, the Netherlands: Freudenthal Group & Freudenthal Institute, Utrecht University and ERME.

Caldeira, A., Viamonte, A. J., Figueiredo, I. & Brás, H. (2020). Using Math Trails as a Travel Guide. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020 (pp. 197-200). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0

Gurjanow, I., & Ludwig, M. (2017). Gamifying math trails with the MathCityMap app: Impact of points and leaderboard on intrinsic motivation. In G. Aldon, & J. Trgalova, *Proceedings of the 13th International Conference on Technology in Mathematics Teaching (ICTMT 13)* (pp. 105-112). Lyon, France.

Gurjanow, I., & Ludwig, M. (2018). The MathCityMap app: A gamified math trail experience. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter, *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education* (pp. 52-53). Umea, Sweden: PME.

Gurjanow, I., & Ludwig, M. (2018). Welchen motivationalen Einfluss hat Gamification auf Neuntklässler bei der Bearbeitung eines Mathtrails? In G. f. Mathematik, *Beiträge zum Mathematikunterricht 2018 (GDMV 3)* (pp. 675-678). Münster: WTM-Verlag.

Gurjanow, I., Taranto, E., Ludwig, M., Alberti, V., & Ferro, R. (2019). Math MOOC UniTo & MathCityMap - Exploring the potentials of a review system in a MOOC environment. In U. T. Jankvist, M. Van den Heuvel-Panhuizen, & M. Veldhuis, *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education (CERME11).* Utrecht, the Netherlands: Freudenthal Group & Freudenthal Institute, Utrecht University and ERME.

Gurjanow, I., Zender, J., & Ludwig, M. (2017). What influences in-service and student teachers use of MathCityMap? In T. Dooley, & G. Gueudet, *Proceedings of the tenth congress of the european society for research in mathematics education (CERME 10)* (S. 2366-2374). Dublin, Ireland.

Gurjanow, I., Zender, J. & Ludwig, M. (2020). MathCityMap - Popularizing Mathematics around the Globe with Math Trails and Smartphone. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), Research on Outdoor STEM Education in the digiTal Age (pp. 103-110). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.13

Jablonski, S. & Ludwig, M. (2020). Development of an Intensive Study Programme on Outdoor Mathematics Teaching with Digital Tools. In M. Ludwig, S. Jablonski, A. Caldeira, &

A. Moura (Eds.), Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020 (pp. 111-118). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.14

Jablonski, S., Ludwig, M., Zender, J. (2018). Task Quality vs. Task Quantity. A dialog-based review system to ensure a certain quality of tasks in the MathCityMap web community. In: H.-G. Weigand, A. Clark-Wilson, A. Donevska-Todorova, E. Faggiano, N. Grønbæk & J. Trgalova (Eds.). *Proceedings of the Fifth ERME Topic Conference (ETC 5) on Mathematics Education in the Digital Age (MEDA) 5-7 September 2018, Copenhagen, Denmark*, S. 115-122.

Ludwig, M., Jablonski, S. (2019). Doing Math Modelling Outdoors – A Special Class Activity designed with MathCityMap. In: *5th International Conference on Higher Education Advances* (HEAd'19). Unversitat Politécnica de Valéncia.

Ludwig, M., Jablonski, S., Gurjanow, I., Baumann-Wehner, M. (2020) MathCityMap live und interaktiv – Mobile mathematische Wanderpfade erstellen und organisieren. In: Tagungsband Wider View.

Zender, J. & Ludwig, M. (2019). The long-term effects of MathCityMap on the performance of German 15 year old students concerning cylindric tasks. In: U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht, the Netherlands: Freudenthal Group & Freudenthal Institute, Utrecht University and ERME.

Others

Amaro, E., Blanco, J. C., Lázaro, C., Recio, T. (2019). MathCityMap: paseos matemáticos a través de dispositivos móviles. Proceedings of the Congreso Iberoamericano La Educación ante el nuevo entorno digital.

Boboňová, I., Čeretková, S. (2019). A Qualitative Study on the Implementation of Digital Technology in Mathematics Education, *INTED2019 Proceedings*, pp. 3647-3651. https://library.iated.org/view/BOBONOVA2019AQU

Čeretková, S., Bulková, K. (2020). Mathematics Trails in Initial Teachers' Education in Slovakia. In: *APLIMAT 2020*, *19th Conference of Applied Mathematics, Book of Abstracts, Slovak University of Technology in Bratislava, 4-6 February 2020, Bratislava, Slovak Republic,* pp. 232 - 237. http://evlm.stuba.sk/APLIMAT/indexe.htm

Espinosa, J. A. (2020). Un paseo por tu ciudad con ojos matemáticos. Uno. Revista de Didáctica de las Matemáticas, (87), 1/2020, pp. 38-42.

Gurjanow, I., Jablonski, S., Ludwig, M. & Zender, J. (2019). Modellieren mit MathCityMap. In: I. Grafenhofer, J. Maaß (Eds.), *Neue Materialien für einen realitätsbezogenen Mathematikunterricht 6. Realitätsbezüge im Mathematikunterricht*. Springer Spektrum, Wiesbaden, pp. 95-105.

Gurjanow, I., Ludwig, M. (2020) Mathematics Trails and Learning Barriers. In: Stillman G., Kaiser G., Lampen C. (eds) Mathematical Modelling Education and Sense-making. International Perspectives on the Teaching and Learning of Mathematical Modelling. Springer, Cham. (pp. 265-275).

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Gurjanow, I., Oliveira, M., Zender, J., Santos, P. A., & Ludwig, M. (2019). Shallow and deep gamification in mathematics trails. In M. Gentile, M. Allegra & H. Söbke (Eds.), *Games and learning Alliance - 2019: 7th International conference, GALA 2018, Palermo, Italy, December 5-7, 2018, Proceedings,* pp. 364-374. Springer International Publishing.

Gurjanow, I., Oliveira, M., Zender, J., Santos, P. A., & Ludwig, M. (2019). Mathematics Trails: Shallow and Deep Gamification. *International Journal of serious games*, vol 6 No 3, 2019.

Jablonski, S. (2019). Problemlösen an der frischen Luft. Mit MathCityMap mathematische Wanderwege erstellen. *Grundschulunterricht Mathematik*, 1/2019.

Jablonski, S., Lazaro, C., Ludwig, M. & Recio, T. (2020). MathCityMap, paseos matemáticos a través de dispositivos móviles. Uno. Revista de Didáctica de las Matemáticas, (87), 1/2020, pp. 47-54.

Ludwig, M., Baumann-Wehner, M., Gurjanow, I., Jablonski, S. (2019). Mathe draußen: MathCityMap. Mit Aufgaben-Wizard und Digitalem Klassenzimmer zum mobilen MathTrail. mathematik lehren, 215, pp. 29-32.

Ludwig, M. & Jablonski, S. (2019). Haciendo matemáticas al aire libre con MathCityMap. In: Jornadas para el Aprendizaje y la Enseñanza de las Matemáticas (19JAEM), 3 a 5 julio 2019, A Coruña, Spain. ISBN: 978-84-948824-4-9.

Ludwig, M., Jablonski, S. (2020). *MathCityMap – Mit mobilen Mathtrails Mathe draußen entdecken. MNU Journal,* pp. 29-36.

Ludwig, M., Jablonski, S. (2020) Mit MathCityMap Mathematik draußen machen – Das Potential mathematischer Wanderpfade als außerschulische Lernorte. In: M. Jungwirth, N. Harsch, Y. Korflür, & M. Stein (Hrsg.): Forschen.Lernen.Lehren an öffentlichen Orten – The Wider View. In: Schriften zur Allgemeinen Hochschuldidaktik. Band 5. Münster: WTM-Verlag 2020, S. 191-196.

Ludwig, M., Jablonski, S., Caldeira, A. & Moura, A. (2020). Research on Outdoor STEM Education in the Digital Age - Background and Introduction. In M. Ludwig, S. Jablonski, A. Caldeira, & A. Moura (Eds.), Research on Outdoor STEM Education in the digiTal Age. Proceedings of the ROSETA Online Conference in June 2020 (pp. 5-9). Münster: WTM. https://doi.org/10.37626/GA9783959871440.0.01

Martínez, J. A. (2020). Si <<miras pero no ves>>, educa tu mirada matemática. Uno. Revista de Didáctica de las Matemáticas, (87), 1/2020, pp. 43-46.

Future research perspectives as the evidence of the MoMaTrE project

Some other research questions, which were posed in the project MoMaTrE proposal require long time research period, but in some project consortium countries, they are going to be investigated. Results of the research are expected within the next years, after the project finalization. This fact is the evidence of the project ideas and activities sustainability and promising perspective for future mathematics (and other school subjects) modern education.

The research questions are: How do pupils and teachers use the material? Do pupils (pre-service teachers, in-service teachers) learn more/better/faster? Are they more motivated to study mathematics? Which topics of mathematics are the most MoMaTrE friendly and why? Does the usage MoMaTrE material and methods increase the self-efficacy of the participants? Does it change mathematical beliefs?

Other research outputs

Final Thesis

Juanes Revuelta, J. (2019, July). TOCAR LAS MATEMÁTICAS EN NUESTRO ENTORNO (TOUCHING MATHS IN OUR ENVIRONMENT)

Ruiz Martín, M. I. (2019, July). Una Semana de Paseos Matematicos y su Aprovechamiento Didáctico (A Week of Mathematical Walks and its Didactic Exploitation)

Vilas Prat, D. (2019, June). Un Paseo Matematico por el Centro Escolar (A Mathematical Walk along the School Center)

Fernández Herrera, M. (2020, July). Un paseo matemático desde el confinamiento

Reginek, P. (2018, May). Förderung des Größenverständnisses bei Schülerinnen und Schülern durch den Einsatz von MathCityMap

Opl, Y. (2018, December). Verwendung und Nutzen der Hinweise der MathCityMap-App- eine empirische Untersuchung

Barlovits, S. (2018, November). Das MathCityMap-Projekt. Eine empirische Untersuchung der Motivation beim Absolvieren eines MathCityMap- Mathtrails

Looß, P. (2018, November). Erweiterung von MathCityMap auf Schulfach Physik – eine Machbarkeitsstudie

Kollas, C. (2019, January). Mathtrails als diagnostisches Mittel zur Lernstandserhebung zu Beginn der Klasse 7

Fricker, C. (2019, July). Fördert das Lernen und Arbeiten außerhalb des Klassenraums die Reaktivierung abgespeicherter Informationen

Hansen-Dörr, A. (2019, June). Wie nutzen Schülerinnen und Schüler der 7. Klasse die Hinweise der MathCityMap-App?

Hummel, F. (June, 2019). Umgang mit sprachlicher Heterogenität im Mathematikunterricht a Beispiel MathCityMap?

Praschak, A. (2019, March). Theoretische und empirische Schwierigkeit von Mathtrailaufgaben unter Berücksichtigung der Lösungsdauer

Liebske, A. (2019, October). Math Trails im Förderschwerpunkt Lernen – Eine Fallstudie in Sek I

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