Generic Tasks: Combinatorics und Probability

Stairs	**How many possibilities exist to go upstairs if one can take one or two steps within each move? The step sequences can be combined.**
 *At the picture, it is important to make clear which part is meant.*  *Further, it is important that one cannot see the total number of steps in the picture.*	<u>Data to be measured:</u> It is necessary to determine the number of steps. It is important that one cannot already see the total number of steps in the picture, because otherwise one does not have to answer the task at the object. <u>Solution:</u> There are different possibilities to solve the task: One possibility is to note down the possibilities systematically. Example n= 5 (1 1 1 1 1) (1 Possibility), (2 1 1 1) (4 Possibilities), (2 2 1) (3 Possibilities), so there are 8 possibilities. Another possibility is to use the Fibonacci numbers. According to this, the number of possibilities to walk a stair with n steps equals the number of possibilities to walk stairs with (n-1) + (n-2) steps. The series would be: (1) 1 2 3 5 8 13 21 34 55 89 etc. <u>Possible Hints:</u> • Note down the possibilities you find to walk upstairs. Further, try to combine different lengths (1 or 2 steps). • Note all combinations systematically. • Systematical notation for 4 steps: 1-1-1-1, 1-1-2, 1-2-1, 2-1-1, 2-2.

Bike Stands

4 bikes should be locked at the stand. Each bike can be locked at the left or right. How many possibilities exist to lock the 5 bikes at the stand? It does not matter whether the bikes are locked "forwards" or "backwards". You can assume that the stand is completely empty.

The picture must not show the total number of bike stands, but nevertheless, it has to describe the object clearly.

This task can also be transferred to seats.

<u>Data to be measured:</u>

One has to count the number of bike stands and whether it is possible to lock one or two bikes at each stand. It is important that one cannot already see the total number of stands in the picture.

<u>Solution:</u>

For the first bike, one has n possibilities, for the second bike n-1 and for bike number k: n-(k+1). These possibilities have to be multiplied. This leads to the following calculation: N= $n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot \big(n-(k+1)\big)$ possibilities.

One can further solve the task with help of the formula of the urn problem „with sequence without repetition". The n balls in the urn symbolize the places at the bike stands. The first bike is assigned to the first drawn ball, the second bike to the second drawn ball, and bike number k to ball number k. The number of possibilities is $N = \frac{n!}{(n-k)!}$

<u>Possible Hints:</u>

- How many possibilities exist for the first bike?
- How many possibilities exist for the second bike?
- Try to find out how one can determine the number of possibilities for two bikes.

Traffic Light

Determine the probability that the traffic light shows green at the moment you arrive. Give the result in percentage.

The traffic light should be easily accessible and visible.

The task can be transferred to bus plans and the question of the probability that one has to wait less than 5 minutes.

Data to be measured:

Period of green t_g= ?

Period of red t_r=?

Important: It has to be a traffic light which cannot be influenced by a button, as the data would not be reproducible. One should observe the clocking and measure the periods at least twice.

Solution:

For the solution of the task one divides the period of green by the total duration of one period consisting of green and red:

$$P(green) = \frac{t_g}{t_g + t_r} \cdot 100\%$$

Possible Hints:

- How do you determine the probability of an event E?
- $P(E) = \frac{favourable\ event}{all\ possible\ events}$
- Determine the length of a period of green and a period of red.